Educational for Drone (eDrone)

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Educational for Drone
(eDrone)
Signal processing and filtering

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Outline

- What is Signal processing
- Analog and digital processing
- Filtering
- Analog Filters
- Digital Filters
- Analog and Digital Filters Comparison
- References

What is Signal processing?

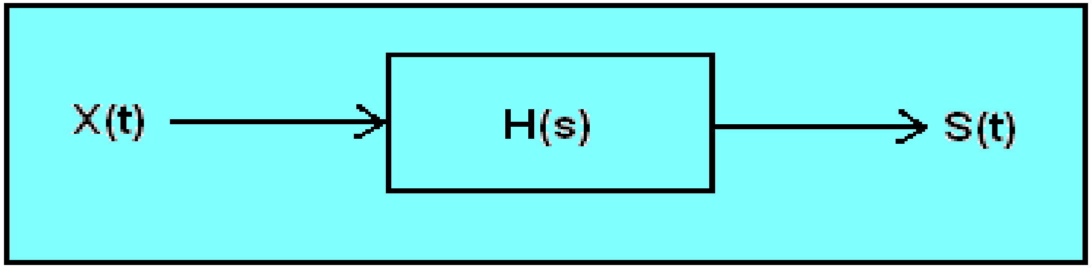
- **Signal processing** is the analysis, interpretation, and manipulation of signals like sound, images time-varying measurement values and sensor data etc…

Need of Signal Processing

- When a signal is transmitted from one point to another there is every possibility of contamination /deformation of the signal by external noise.
- To retrieve the original signal at the receiver, suitable filters are to be used. i.e the signal is processed to obtain the pure signal.

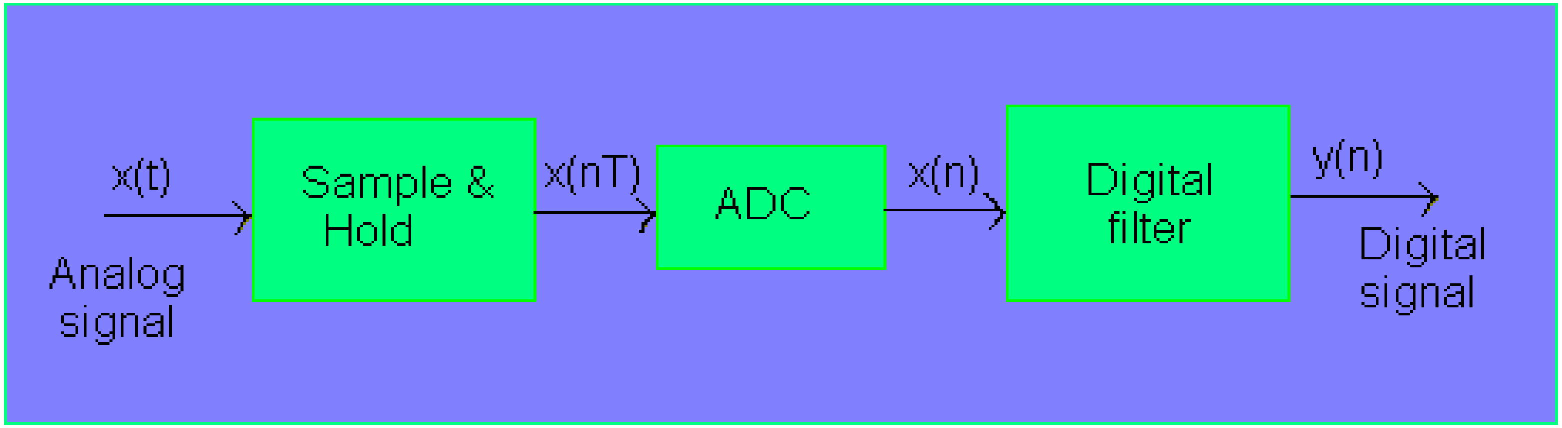
Categories of signal processing - analog

- It is used for signals that have not been digitized.
- Involves linear electronic circuits such as passive filters, active filters, additive mixers, integrators and delay lines.
- Involves non-linear circuits such as compandors, multiplicators (frequency mixers and voltage-controlled amplifiers), voltage-controlled filters, voltage-controlled oscillators and phase-locked loops.



Categories of signal processing - digital

- It is used for signals that have been digitized, processing is done by general-purpose computers or by digital circuits such as ASICs, field-programmable gate arrays or specialized digital signal processors (DSP chips).



Advantages of Digital over analog signal processing

- **Accuracy:** The analog circuits are prone to temperature and external effects, but the digital filters have no such problems.
- **Flexibility:** Reconfiguration of analog filters is very complex whereas the digital filters can be reconfigured easily by changing the program coefficients.
- Digital signals can be easily stored on any magnetic media or optical media are using semiconductor chips.
- **Easy operation:** Even complex mathematical operations can be performed easily using computers, which is not the case with analog processing.
- **Multiplexing:** Digital signal processing provides the way for Integrated service digital network (ISDN) where digitized signals can be multiplexed with other digital data and transmitted through the same channel.

Limitations in digital signal processing

- Bandwidth restrictions
- Speed limitations
- Finite word length problems.

Filtering

- Filtering is one of the most widely used complex signal processing operations
- The system implementing this operation is called a filter
- A filter passes certain frequency components without any distortion and blocks other frequency components

Filtering

- The range of frequencies that is allowed to pass through the filter is called the passband, and the range of frequencies that is blocked by the filter is called the stopband
- In most cases, the filtering operation for analog signals is linear

Filter uses

- Signal separation: is needed when a signal has been contaminated with interference, noise or other signals.

- Signal restoration: is used when a signal has been distorted in some way or other.

Examples

- an audio recording made with poor equipment may be filtered to get the “original” sound.
- deblurring of an image occurred with an improperly focused lens or a shaky camera.

NOTE

- these problems can be solved with either analog or digital filters!

Filters

Filters may be classified as either digital or analog.

- **Analog filters** may be classified as either passive or active and are usually implemented with R, L, and C components and operational amplifiers.
- **Digital filters** are implemented using a digital computer or special purpose digital hardware.

Analog filters

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Analog filter

- Analog filters are cheap and have a large dynamic range in both amplitude and frequency. But in terms of performance they are not superior to digital filters.

Active V.S. Passive

- An active filter is one that, along with R, L, and C components, also contains an energy source, such as that derived from an operational amplifier.
- A passive filter is one that contains only R, L, and C components. It is not necessary that all three be present. L is often omitted (on purpose) from passive filter design because of the size and cost of inductors – and they also carry along an R that must be included in the design.

Analysis V.S. Synthesis

- The analysis of analog filters is well described in filter text books. The most popular include Butterworth, Chebyshev and elliptic methods (their difference will be discussed later).

- The synthesis (realization) of analog filters, that is, the way one builds (topological layout) the filters, received significant attention during 1940 thru 1960. Leading the work were Cauer and Tuttle. Since that time, very little effort has been directed to analog filter realization.

Filtering operation

- The filtering operation of a linear analog filter is described by the convolution integral

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$$

where $x(t)$ is the input signal, $y(t)$ is the output of the filter, and $h(t)$ is the impulse response of the filter

Filtering

- A lowpass filter passes all low-frequency components below a certain specified frequency f_c , called the cutoff frequency, and blocks all high-frequency components above f_c
- A highpass filter passes all high-frequency components a certain cutoff frequency and blocks all low-frequency components below f_c

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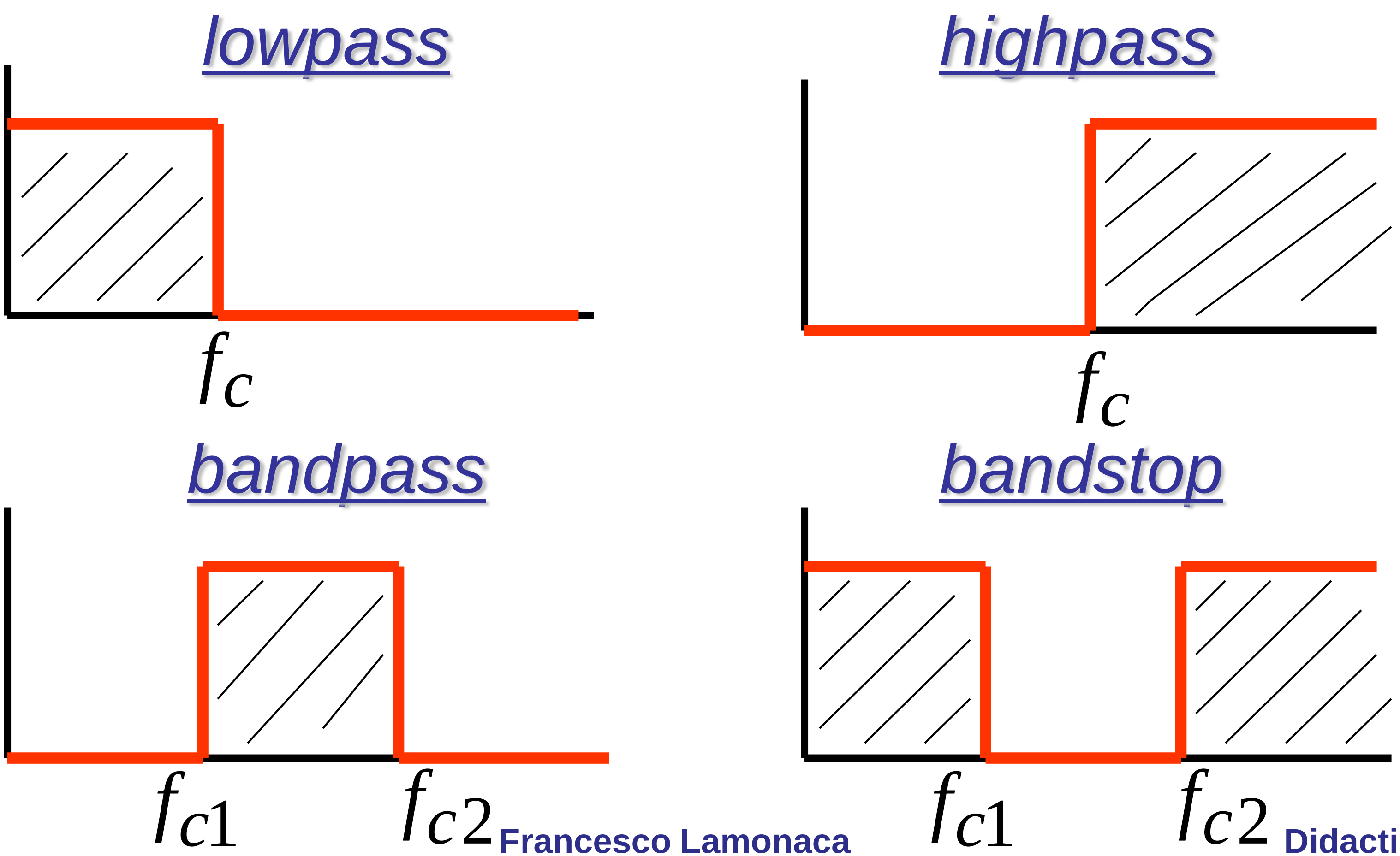
Filtering

- A bandpass filter passes all frequency components between 2 cutoff frequencies, f_{c1} and f_{c2}, where $f_{c1} < f_{c2}$, and blocks all frequency components below the frequency f_{c1} and above the frequency f_{c2}
- A bandstop filter blocks all frequency components between 2 cutoff frequencies, f_{c1} and f_{c2}, where $f_{c1} < f_{c2}$, and passes all frequency components below the frequency f_{c1} and above the frequency f_{c2}

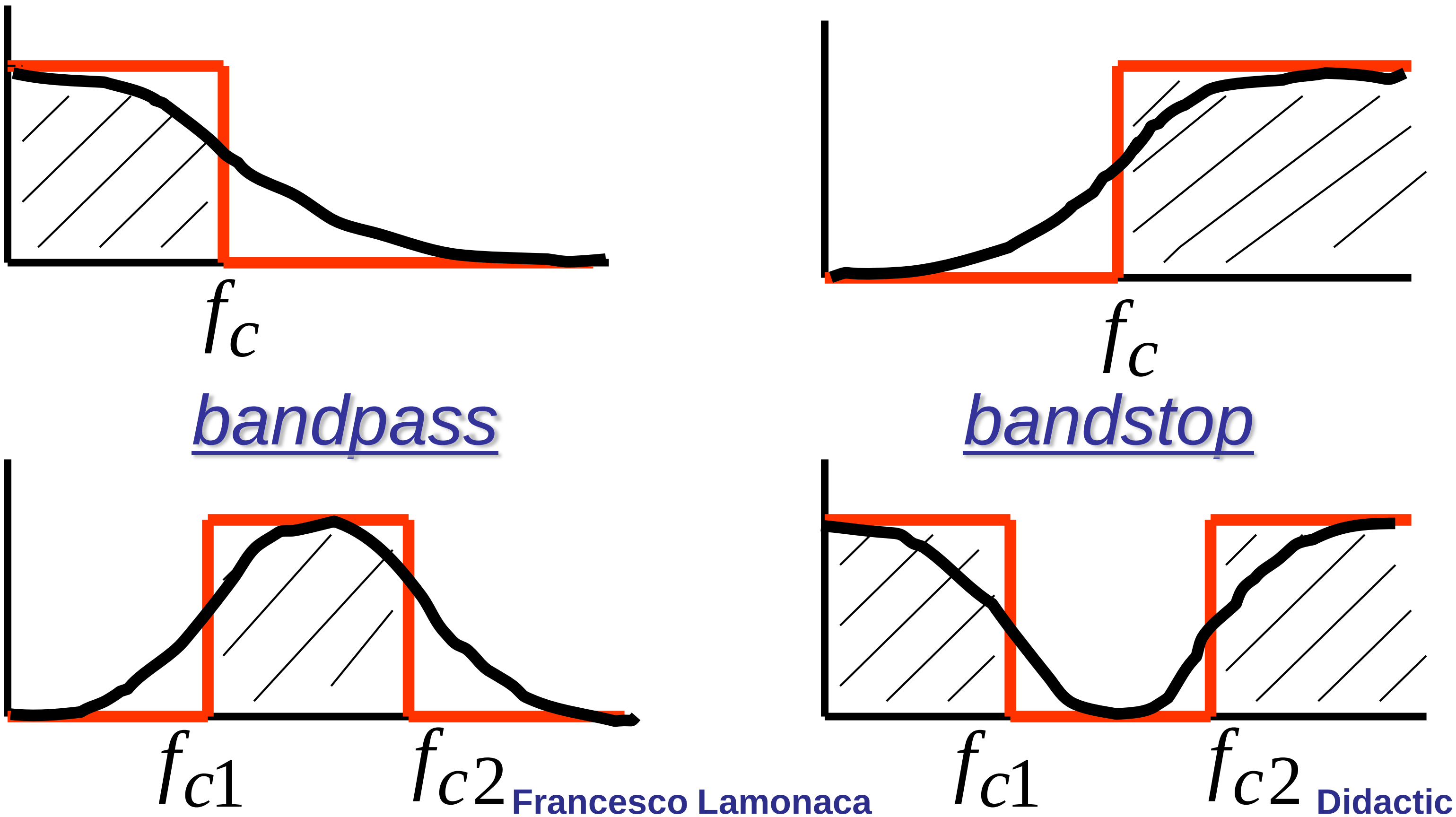
Passive analog filters

- Ideal characteristics



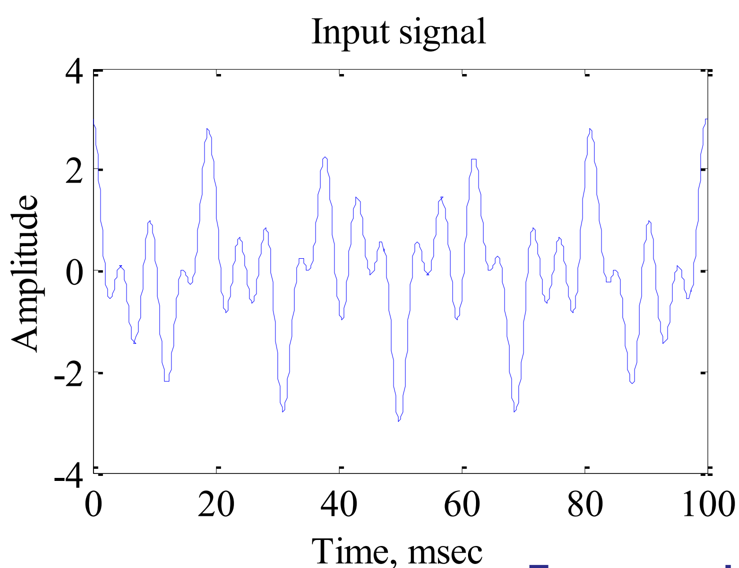
Passive analog filters

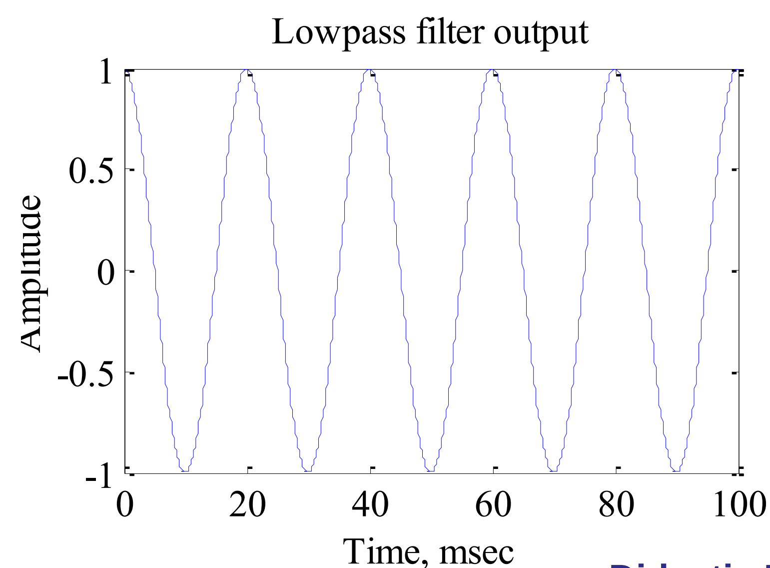
- Realistic Filters



Example of Filtering

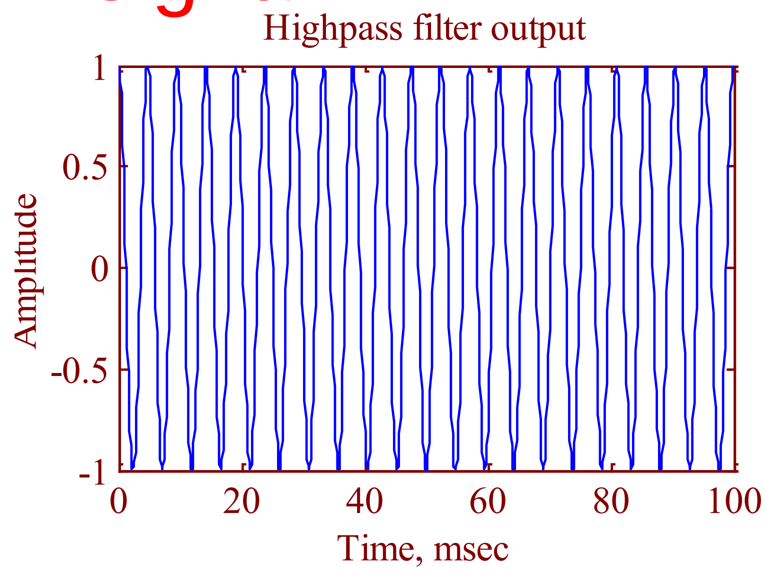
- Figures below illustrate the lowpass filtering of an input signal composed of 3 sinusoidal components of frequencies 50 Hz, 110 Hz, and 210 Hz

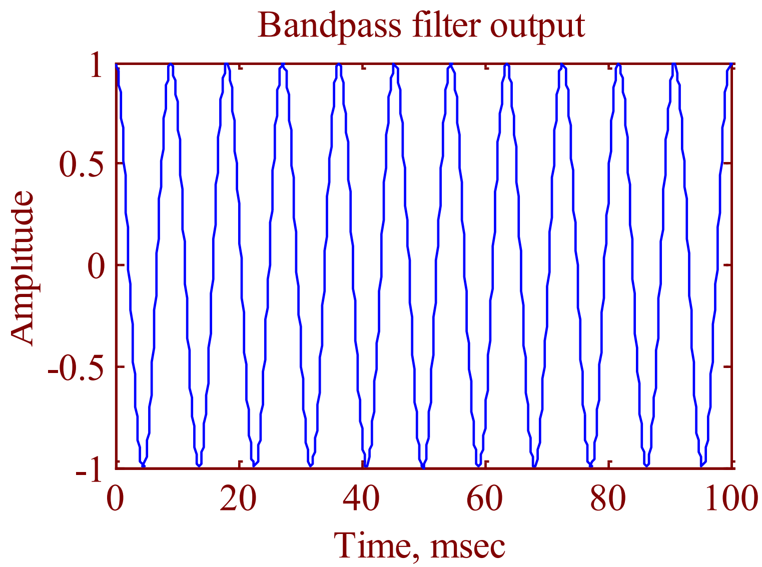




Example of Filtering

- Figures below illustrate highpass and bandpass filtering of the same input signal





Other types of filters

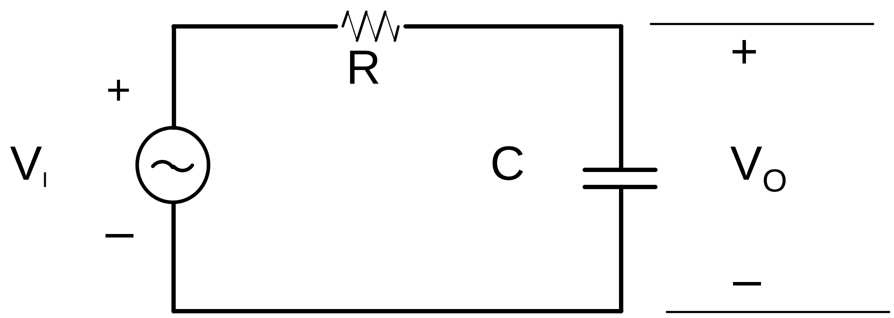
- There are various other types of filters
- A filter blocking a single frequency component is called a notch filter
- A multiband filter has more than one passband and more than one stopband
- A comb filter blocks frequencies that are integral multiples of a low frequency

Filtering Example 2

- In many applications the desired signal occupies a low-frequency band from dc to some frequency f_L Hz, and gets corrupted by a high-frequency noise with frequency components above f_H Hz with $f_H > f_L$
- In such cases, the desired signal can be recovered from the noise-corrupted signal by passing the latter through a lowpass filter with a cutoff frequency f_c where

$$f_L < f_c < f_H$$

Passive low pass filter



Low pass filter circuit

$$\frac{V_O(jw)}{V_i(jw)} = \frac{\frac{1}{jwC}}{R + \frac{1}{jwC}} = \frac{1}{1 + jwRC}$$

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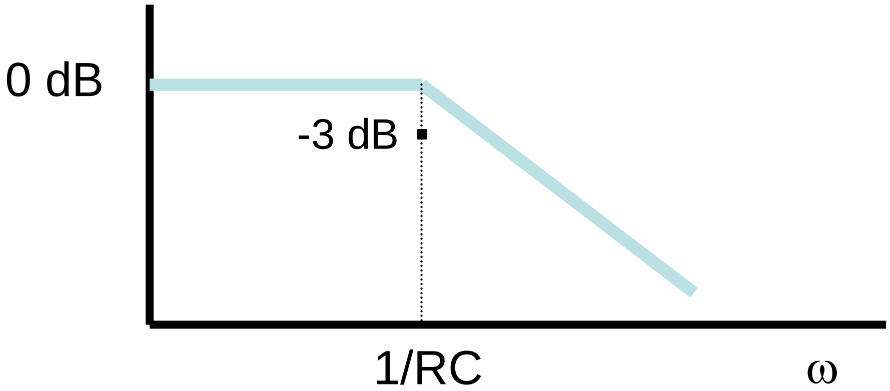
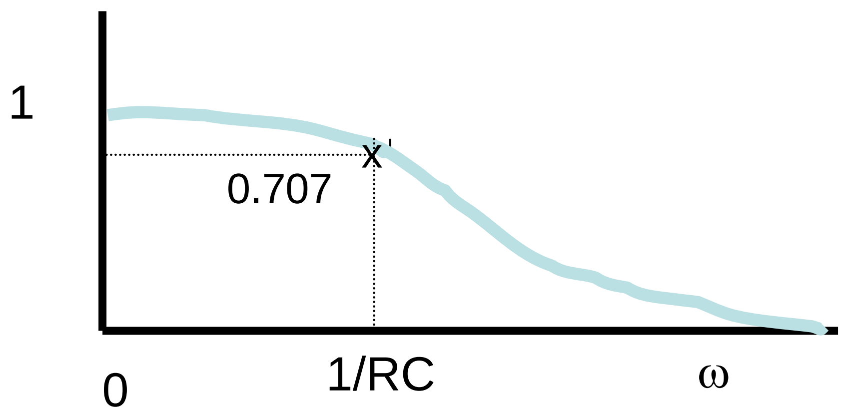
Characteristic of a low pass filter



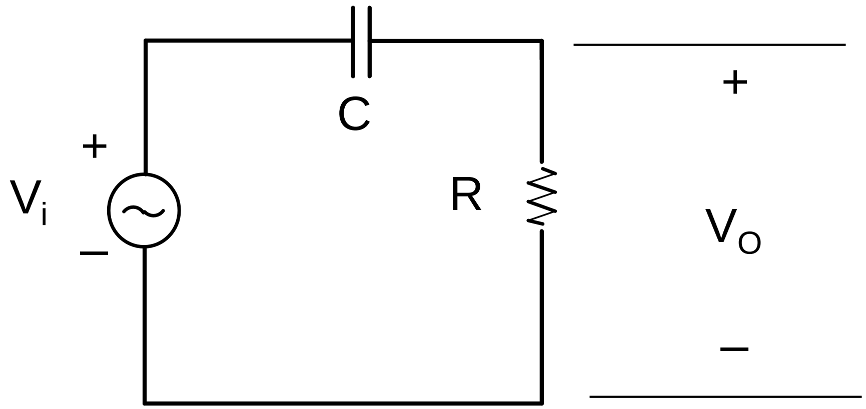
Bode

Passes low frequencies
Attenuates high frequencies



Linear Plot

Passive high pass filter

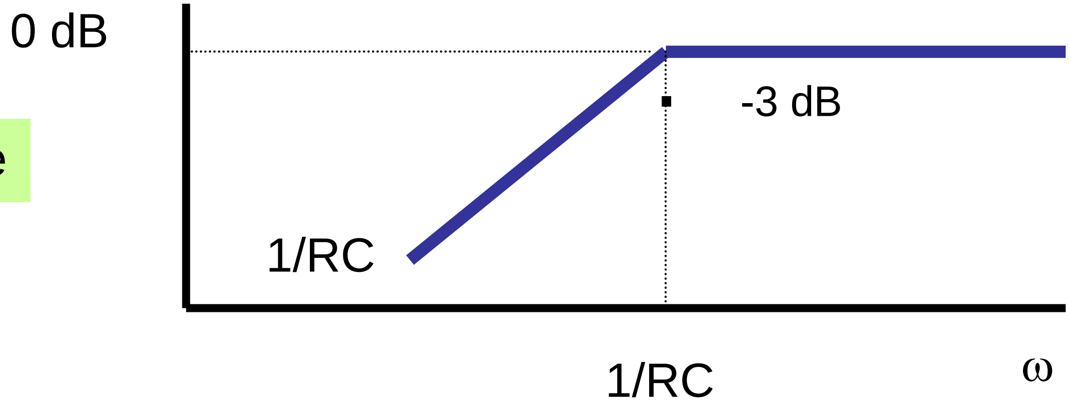


High Pass Filter

$$\frac{V_O(jw)}{V_i(jw)} = \frac{R}{R+\frac{1}{jwC}} = \frac{jwRC}{1+jwRC}$$

Passive high pass filter characteristic

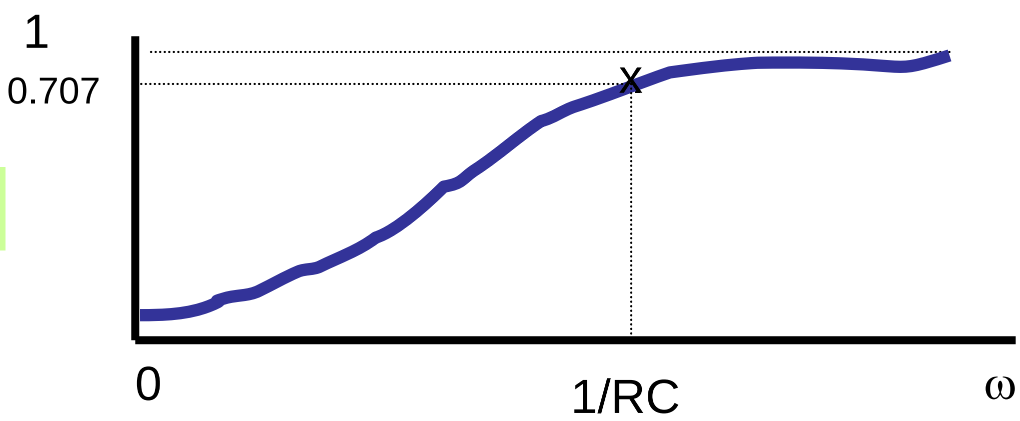
Bode



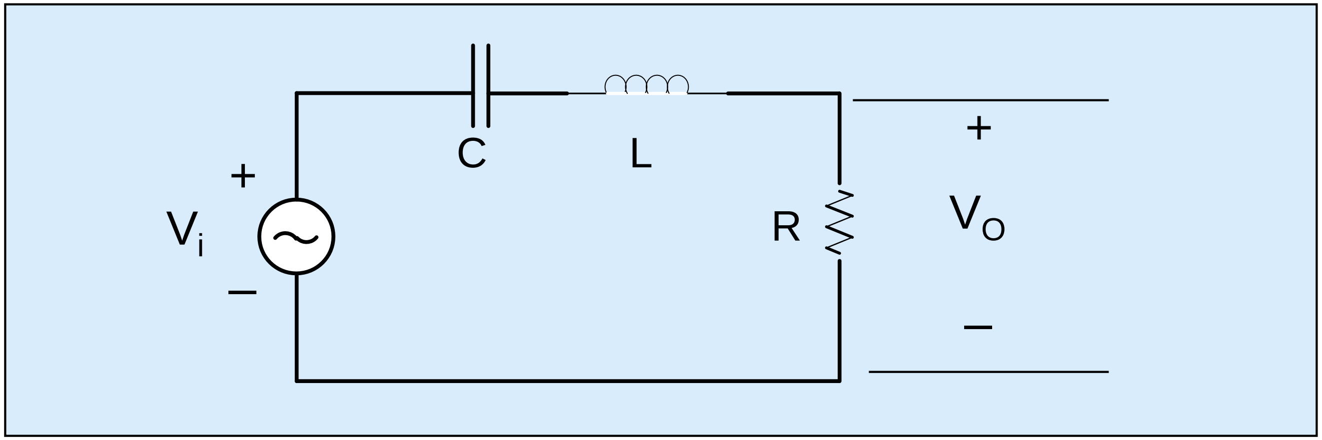
Passes high frequencies

Attenuates low frequencies

Linear



Passive band pass filter

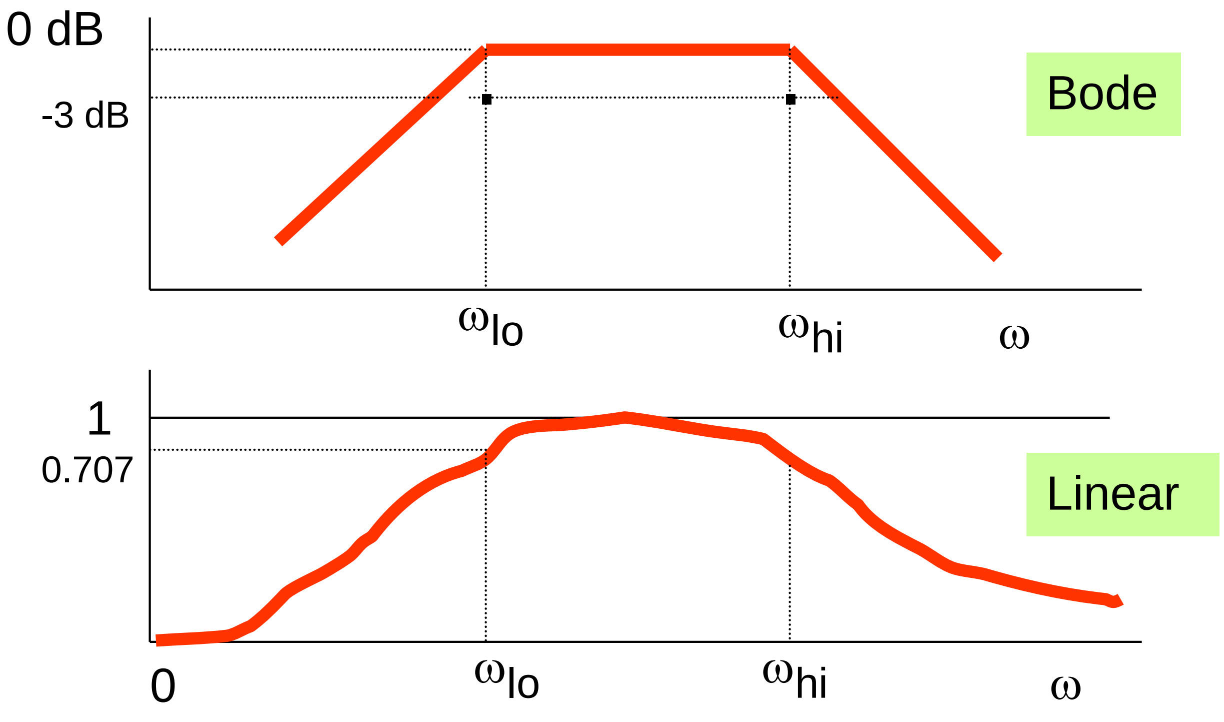


When studying series resonant circuit we showed that;

$$\frac{V_O(s)}{V_i(s)} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Passive band pass filter characteristic

We can make a bandpass from the previous equation and select the poles where we like. In a typical case we have the following shapes.



Example

Suppose we use the previous series RLC circuit with output across R to design a bandpass filter. We will place poles at –200 rad/sec and – 2000 rad/sec hoping that our –3 dB points will be located there and hence have a bandwidth of 1800 rad/sec. To match the RLC circuit form we use:

$$\frac{2200\,s}{s^2+2200\,s+400000}=\frac{2200s}{(s+200)(s+2000)}=\frac{2200\,s}{200x2000(1+\frac{s}{200})(1+\frac{s}{2000})}$$

The last term on the right can be finally put in Bode form as;

$$\frac{0.0055\,jw}{(1+\frac{jw}{200})(1+\frac{jw}{2000})}$$

Example

From this last expression we notice from the part involving the zero we have in dB form;

20log(.0055) + 20logw

Evaluating at w = 200, the first pole break, we get a 0.828 dB what this means is that our –3dB point will not be at 200 because we do not have 0 dB at 200. If we could lower the gain by 0.829 dB we would have – 3dB at 200 but with the RLC circuit we are stuck with what we have. What this means is that the – 3 dB point will be at a lower frequency. We can calculate this from

$$\log\frac{200}{wlow}x20\frac{dB}{dec} = 0.828\,dB$$

Example

This gives an w_{low} = 182 rad/sec. A similar thing occurs at w_{hi} where the new calculated value for w_{hi} becomes 2200. These calculations do no take into account a 0.1 dB that one pole induces on the other pole. This will make w_{lo} somewhat lower and w_{hi} somewhat higher.

One other thing that should have given us a hint that our w_1 and w_2 were not going to be correct is the following:

$$\frac{\frac{R}{L}s}{(s^2+\frac{R}{L}s+\frac{1}{LC})} = \frac{(w_1+w_2)s}{(s^2+(w_1+w_2)s+w_1 w_2)}$$

What is the problem with this?

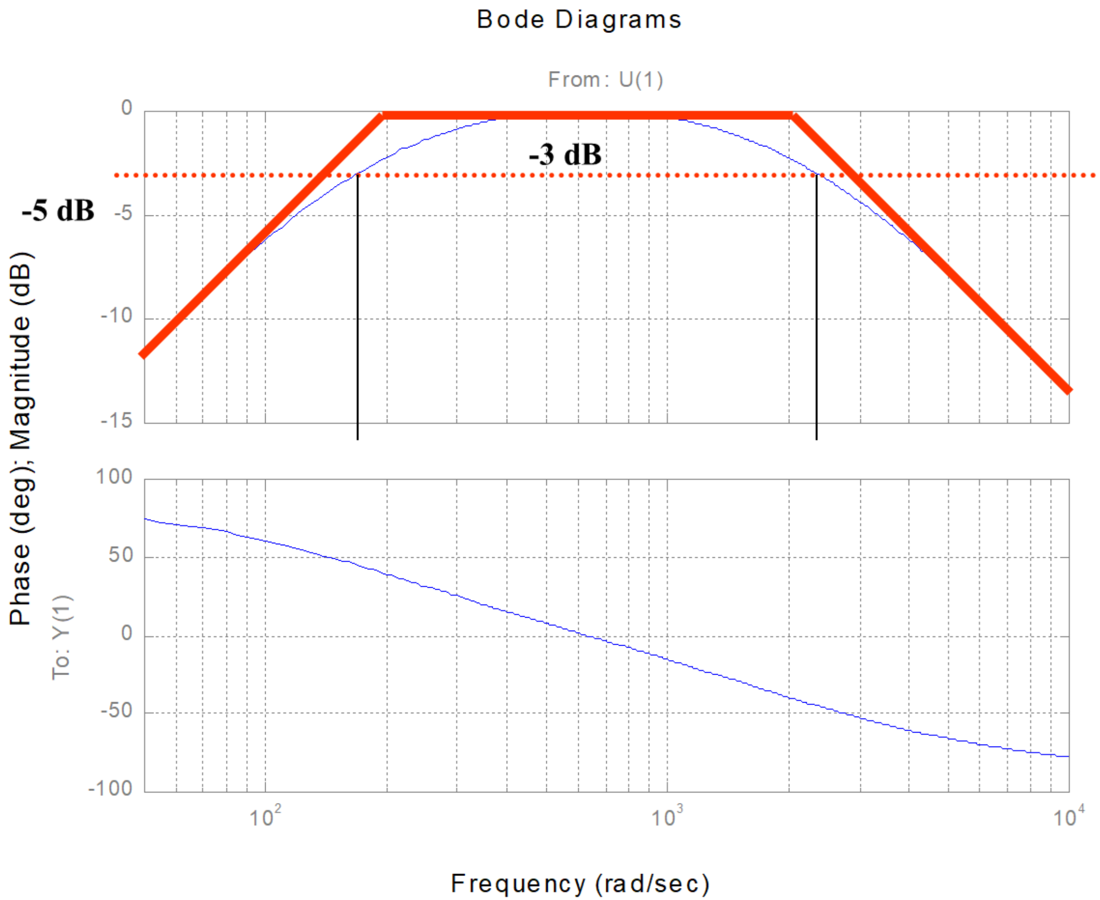
Example

The problem is that we have

$$\frac{R}{L} = \left(w_1 + w_2\right) = BW = \left(w_2 - w_1\right)$$

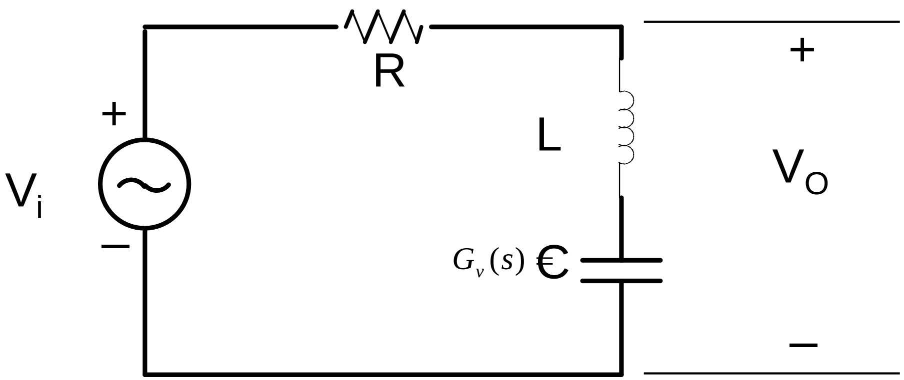
Therein lies the problem. Obviously the above cannot be true and that is why we have aproblem at the –3 dB points.

We can write a Matlab program and actually check all of this.
We will expect that w_1 will be lower than 200 rad/sec and w_2 will be higher than 2000 rad/sec.



Bode Diagrams
From: U(1)
-3 dB
-5 dB
0
-5
-10
-15
100
50
0
-50
-100
Phase (deg); Magnitude (dB)
To: Y(1)
10^2
10^3
10^4
Frequency (rad/sec)

Passive Band stop Filter



The transfer function for V_O/V_i can be expressed as follows:

$$G_v(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

- Comments

This is of the form of a band stop filter. We see we have complex zeros on the jw axis located

$$\pm j\frac{1}{\sqrt{LC}}$$

From the characteristic equation we see we have two poles. The poles an essentially be placed anywhere in the left half of the s-plane. We see that they will be to the left of the zeros on the jw axis.

We now consider an example on how to use this information.

- example

Design a band stop filter with a center frequency of 632.5 rad/sec and having poles at –100 rad/sec and –3000 rad/sec.

The transfer function is:

$$\frac{s^2 + 300000}{s^2 + 3100s + 300000}$$

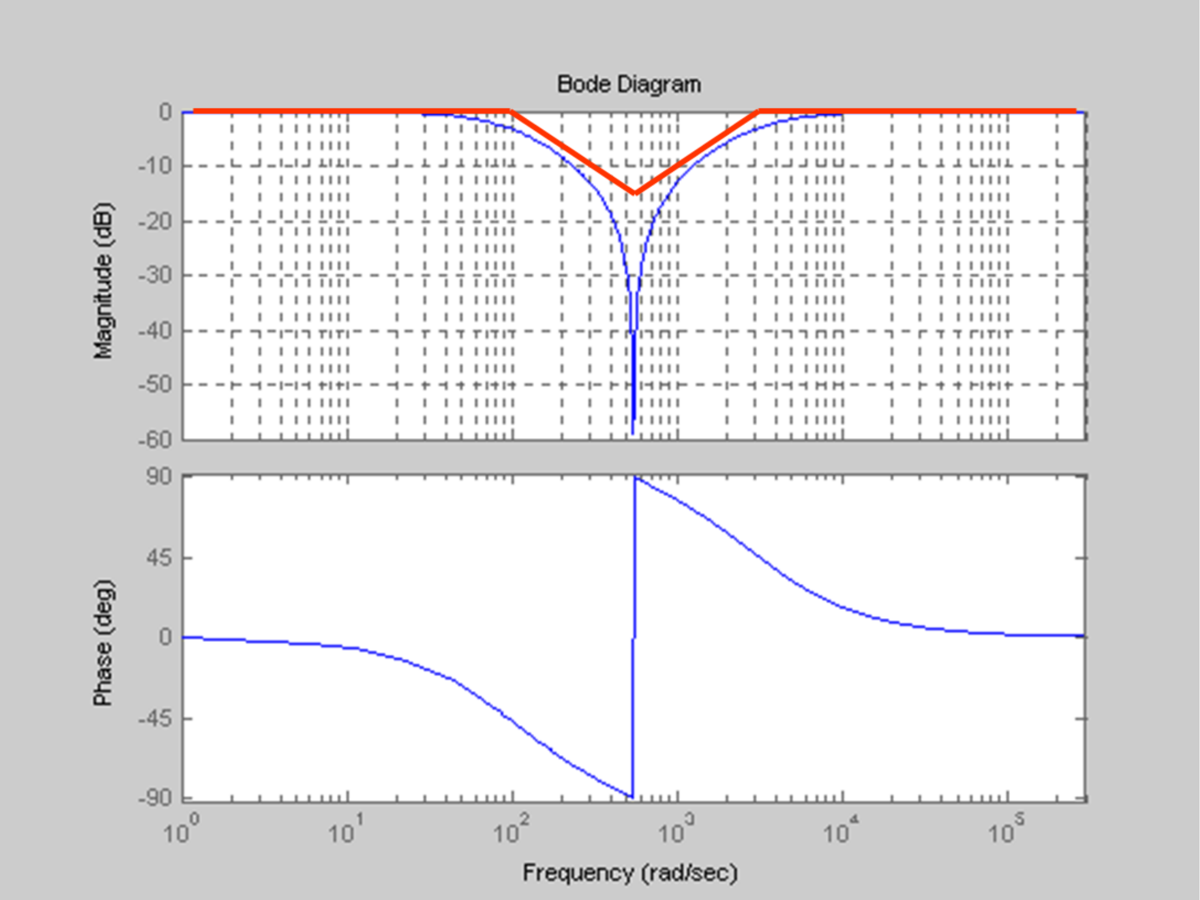
We now write a Matlab program to simulate this transfer function.

num = [1 0 300000];

den = [1 3100 300000];

w = 1 : 5 : 10000;

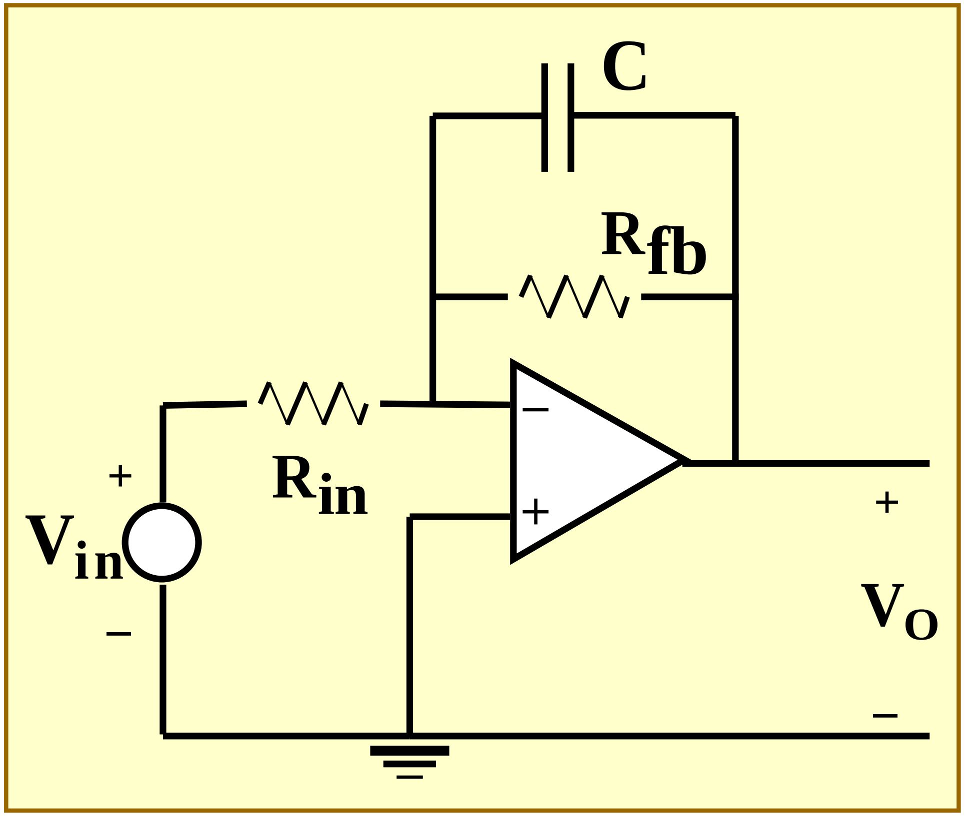
Bode(num,den,w)



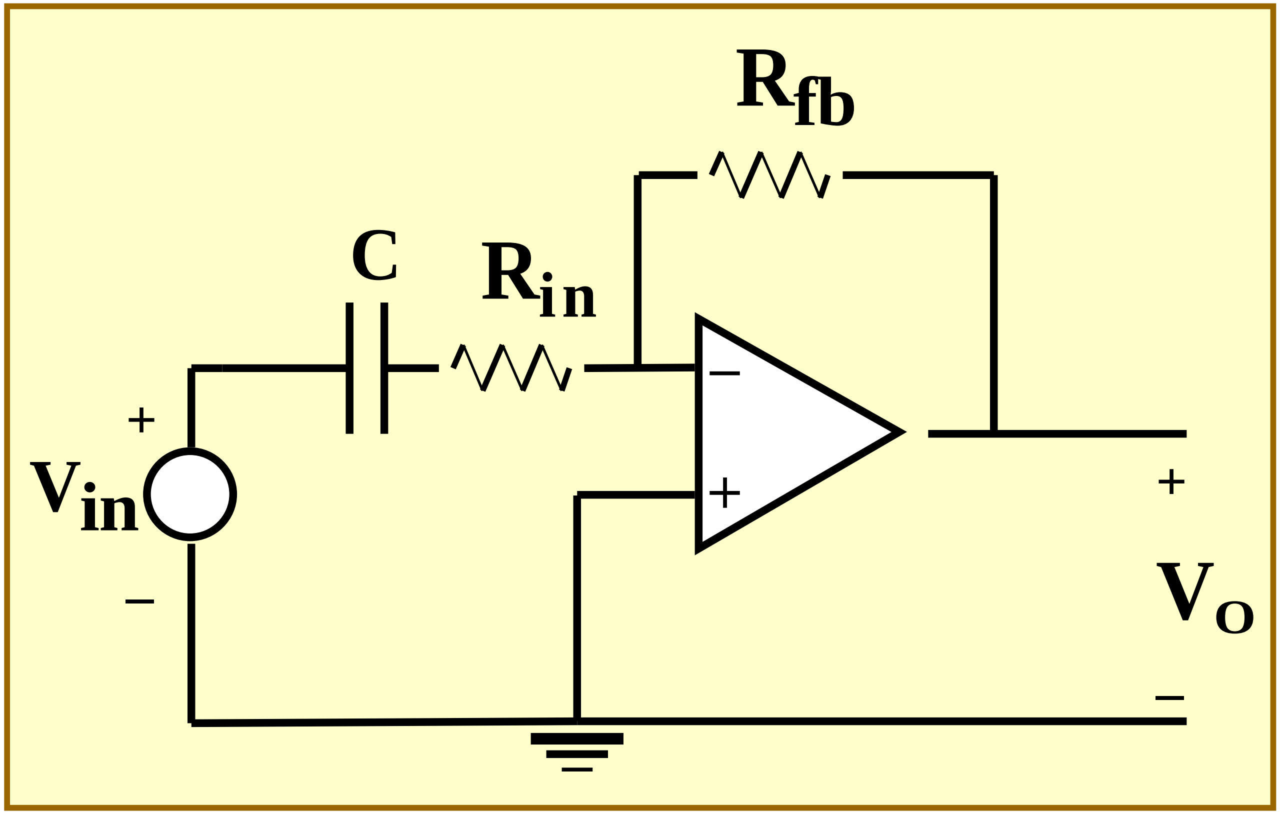
Bode

Matlab

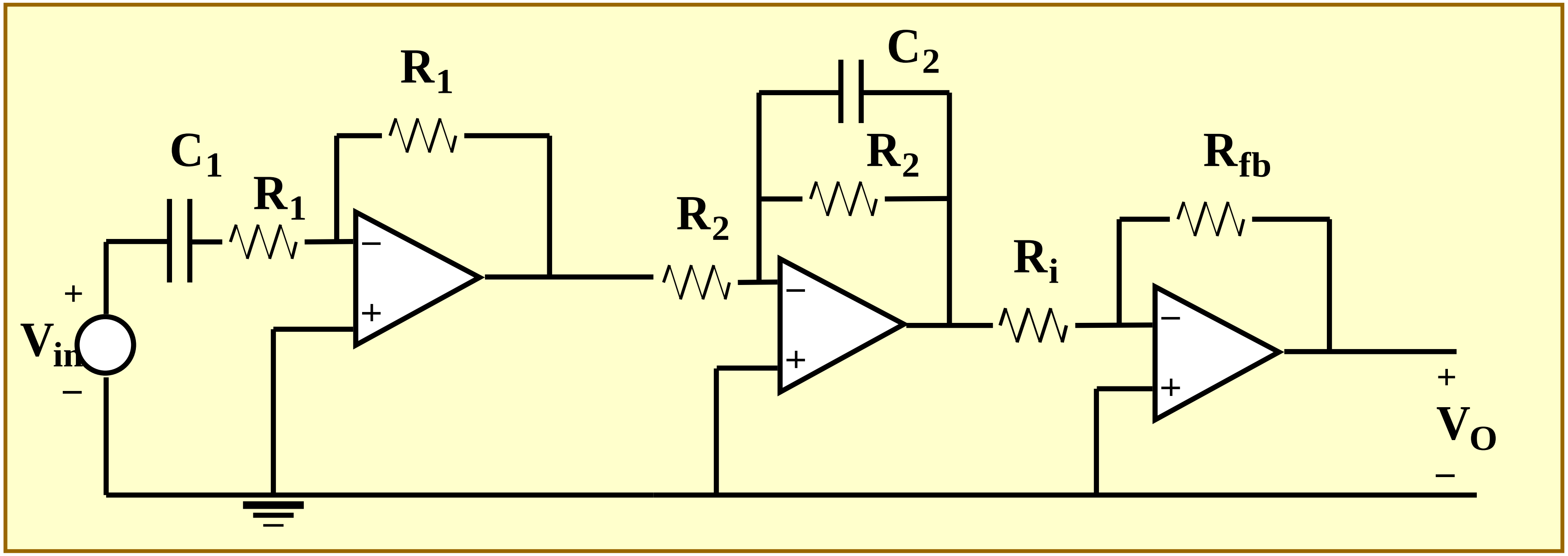
Active low pass filter



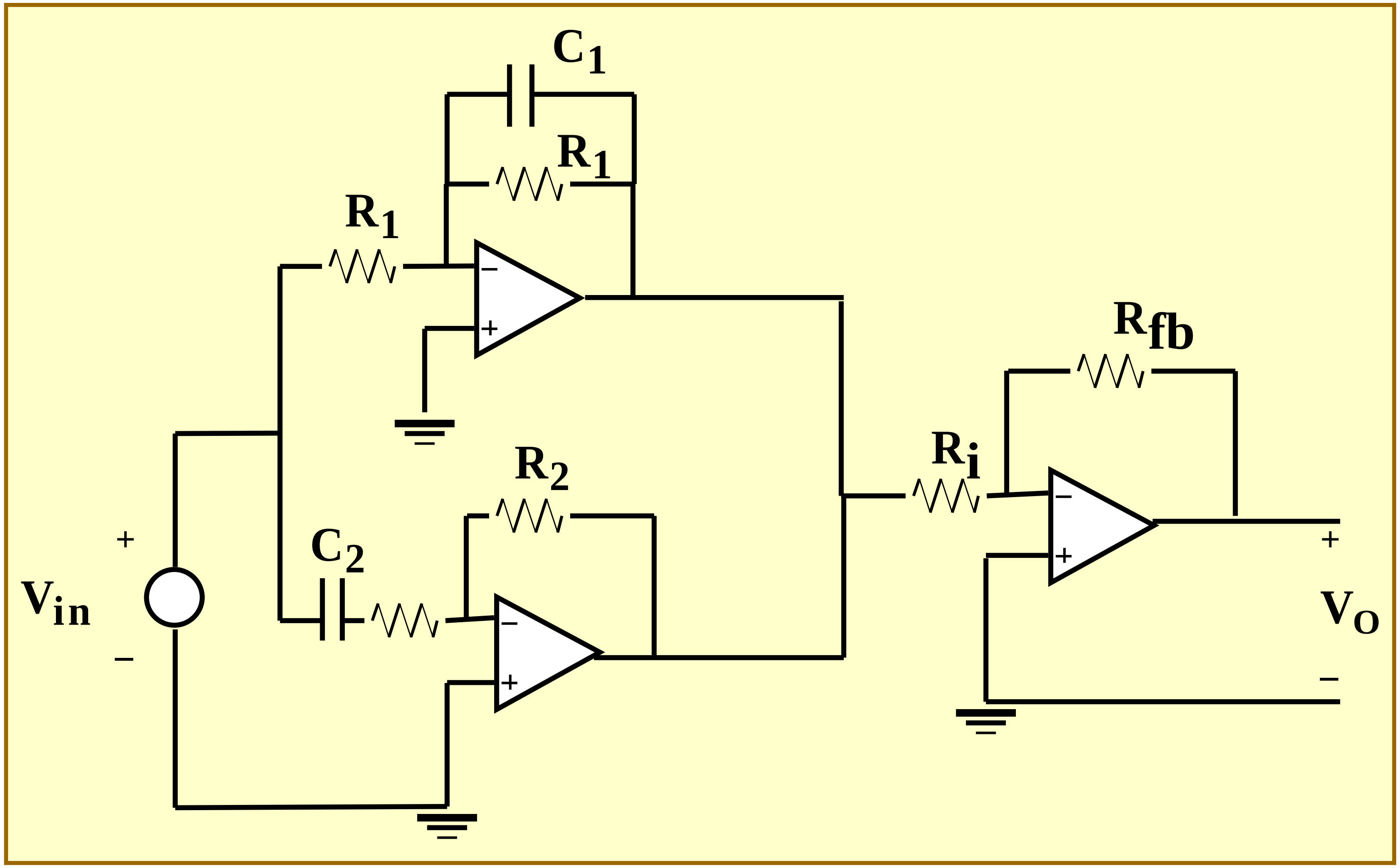
Active high pass filter



Active band pass filter



Active band stop filter

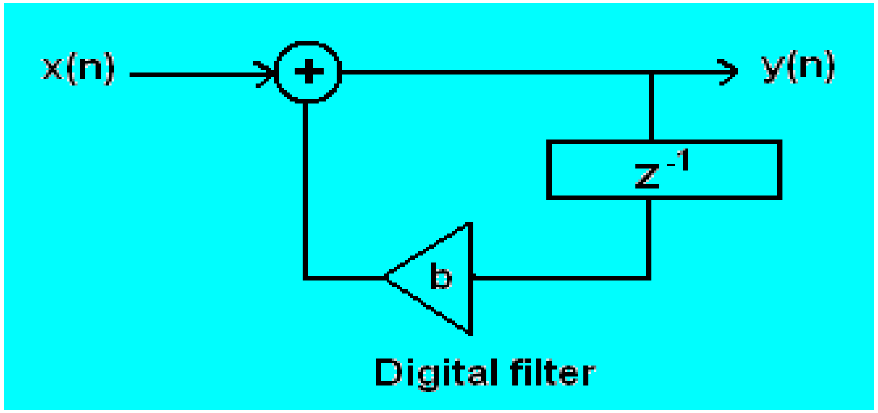


Digital filters

- Generally speaking, digital filters have become the focus of attention in the last 40 years. The interest in digital filters started with the advent of the digital computer, especially the affordable PC and special purpose signal processing boards. People who led the way in the work (the analysis part) were Kaiser, Gold and Radar.
- A digital filter is simply the implementation of an equation(s) in computer software. There are no R, L, C components as such. However, digital filters can also be built directly into special purpose computers in hardware form. But the execution is still in software.

Digital filters

- A digital filter processes and generates digital data.
- A digital filter constitutes elements like adder, multiplier and delay units.
- Digital filters are vastly superior in the level of performance in comparison to analog filters.



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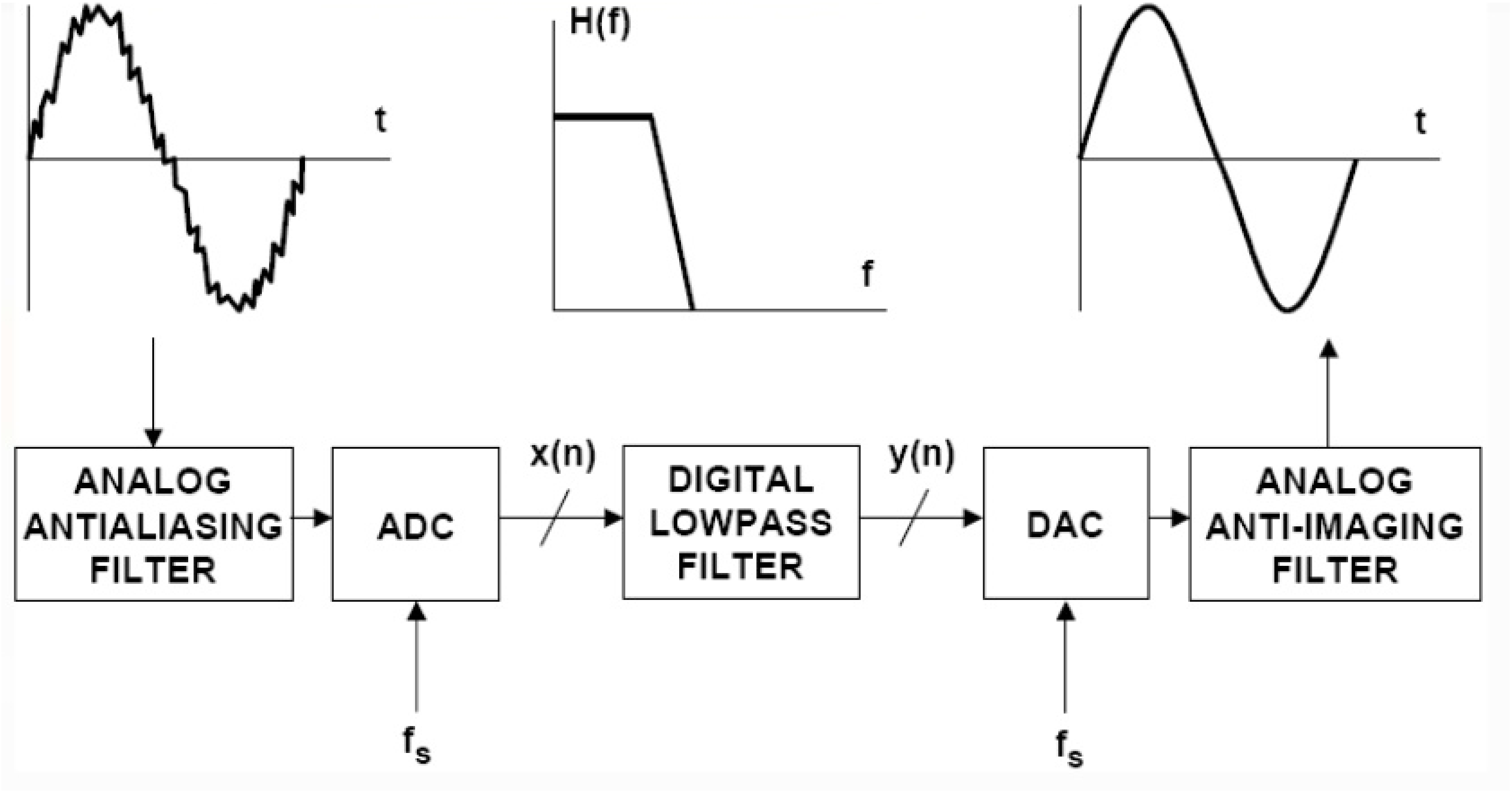


t
H(f)
f
t
ANALOG ANTIALIASING FILTER
ADC
x(n)
DIGITAL LOWPASS FILTER
y(n)
DAC
ANALOG ANTI-IMAGING FILTER
f_s
f_s

Advantages

- Unlike analog filters ,the digital filter performance is not influenced by component ageing, temperature and power variations.
- A digital filter is highly immune to noise and relatively stable.
- Digital filters afford a wide variety of shapes for the amplitude and phase responses.
- Impedance matching problems are minimum.
- Transportation and reconfiguration is very easy, which is not true in the case of analog filters.
- Multiple filtering is possible only in digital filters.
- Computational problems are minimum.

Disadvantages

- Quantization error occurs due to finite word length in the representation of signals and parameters.
- Digital filters also suffer from Bandwidth problems.

DIGITAL FILTERS	ANALOG FILTERS
High Accuracy	Less Accuracy - Component Tolerances
Linear Phase (FIR Filters)	Non-Linear Phase
No Drift Due to Component Variations	Drift Due to Component Variations
Flexible, Adaptive Filtering Possible	Adaptive Filters Difficult
Easy to Simulate and Design	Difficult to Simulate and Design
Computation Must be Completed in Sampling Period - Limits Real Time Operation	Analog Filters Required at High Frequencies and for Anti-Aliasing Filters
Requires High Performance ADC, DAC & DSP	No ADC, DAC, or DSP Required

Differences between analog and digital filters :

- An analog filter is constructed using active, passive components like resistors, capacitors and op amps etc..
- A digital filter constitutes adder, multiplier and delay elements
- An analog filter is denoted by a differential equation.
- A digital filter is denoted by a difference equation.
- Laplace transform is used for the analysis of analog filter.
- Z transforms are used for the analysis of digital filters.
- The frequency response of an analog filter can be modified by changing the components.
- The frequency response can be changed by changing the filter coefficients.

Types of Digital filters

- FIR Filters(Finite impulse response filters)
- IIR Filters (Infinite Impulse response filters)

FIR Filters

- The digital filter whose impulse response is of finite duration is known as Finite impulse response filter. The response of the FIR filter depends only on the present and past input samples.
- These FIR filters are also called non recursive filters.
- So, in FIR the impulse sequence is of finite duration, i.e. it has a finite number of non-zero terms.

The system with the impulse response

$$h(n) = \begin{cases} 2 & n \le 4 \\ 0 & otherwise \end{cases}$$

denotes an FIR system.

Example

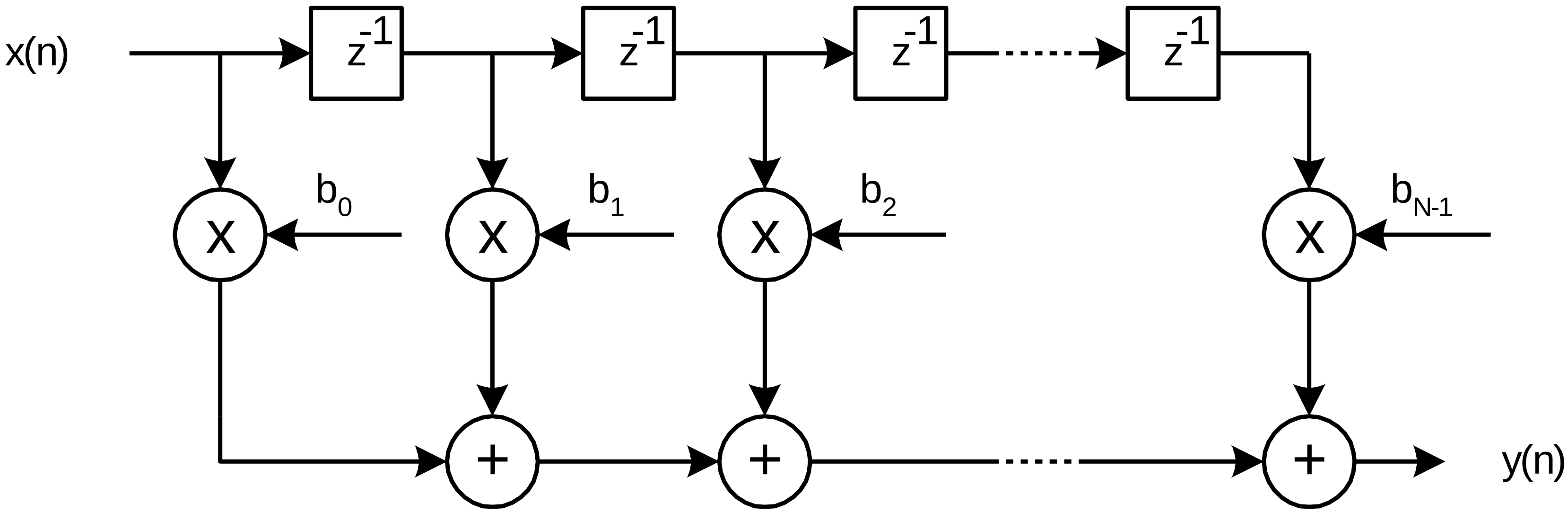
- The following difference equation denotes the finite impulse response filter.

$$y(n) = \sum_{i=0}^{N-1} b_i \; x(n-i)$$

- For i<0, y(n)=0 i.e. the impulse response is finite and it exists only for n>0

FIR realization

$$y(n) = \sum_{i=0}^{N-1} b_i \; x(n-i)$$ **FIR equation**



Filter structure

Advantages of FIR Filters

- FIR filters can be designed with exact linear phase. These linear phase filters are important for applications where frequency dispersion due to non-linear phase is hazardous. (For example speech processing and data transmission)
- FIR filters are stable
- Round off noise can be eliminated in FIR filters
- FIR filters can be efficiently implemented in multirate DSP systems
- FIR filters reduce the computation complexity

Disadvantages of FIR Filters

- As large number of impulse response samples are required to properly approximate sharp cutoff FIR filters the processing will become complex due to slow convolution.
- The delay of linear phase FIR filters can sometimes create problems in some DSP applications.

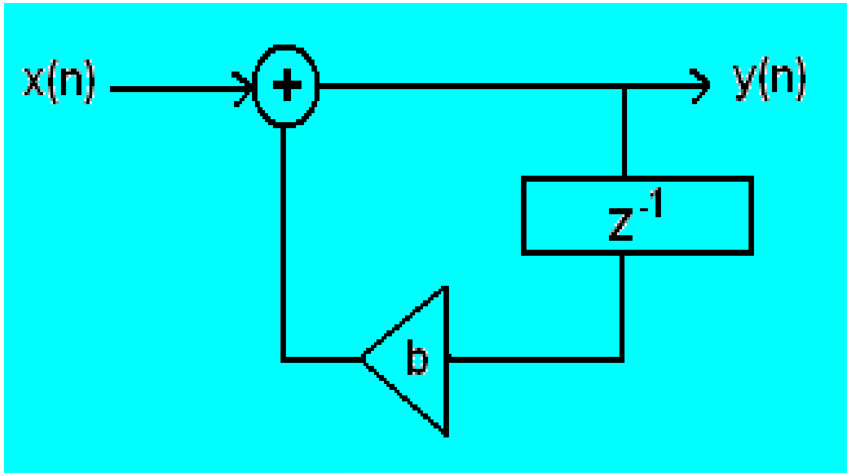
IIR Filters

- The digital filter whose impulse response is of infinite duration is known as Infinite impulse response filter.
- The response of an IIR filter is a function of current and past input signal samples and past output signal samples.
- It is also called recursive filter.

Example

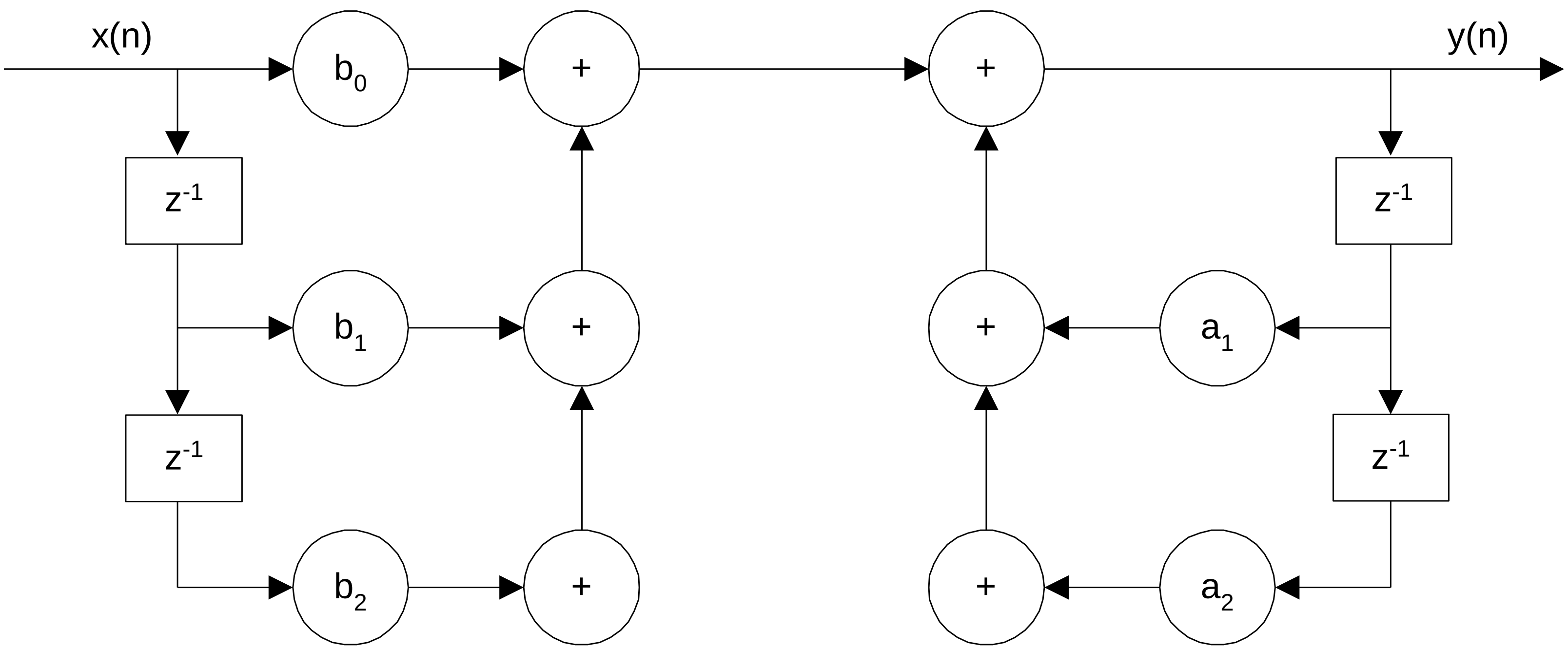
- A simple first order difference equation illustrates the IIR filter.

$$y(n) = x(n) + b \,.\, y(n-1)$$



IIR Equation and an Example

$$y[n] = \sum_{k=0}^{N} b[k]x[n-k] + \sum_{k=1}^{M} a[k]y[n-k]$$ **IIR Equation**



IIR Structure in the case N = M = 2

Advantages of IIR filters

- An IIR filter has lesser number of side lobes in the stop-band than an FIR filter with the same number of parameters.
- Also the implementation of an IIR filter involves fewer parameters, less memory requirements and lower computational complexity.

Advantages of IIR filters

- ✓ IIR filters do not have linear phase and also they are not very stable.
- ✓ Realization of IIR filters is not very easy as compared to FIR filters
- ✓ As it is a recursive filter the number of coefficients is very large and the memory requirements are also high

Digital Filter Specifications

- Usually, either the magnitude and/or the phase (delay) response is specified for the design of digital filter for most applications
- In some situations, the unit sample response or the step response may be specified
- In most practical applications, the problem of interest is the development of a realizable approximation to a given magnitude response specification

Digital Filter Specifications

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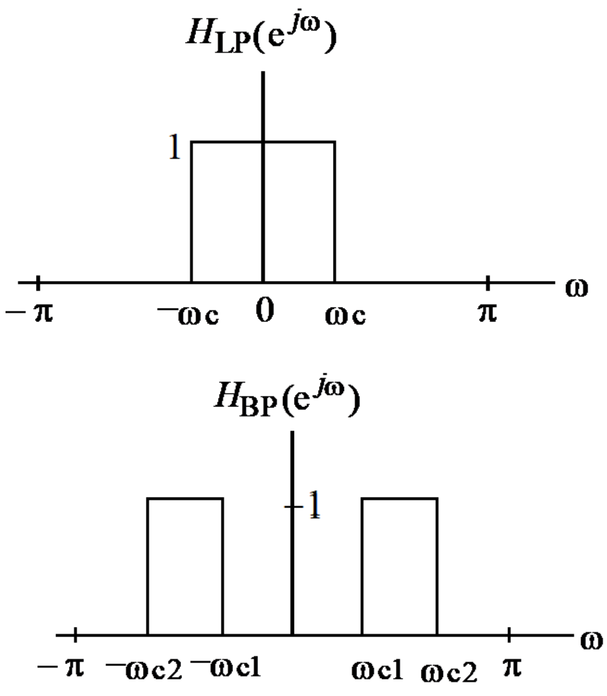
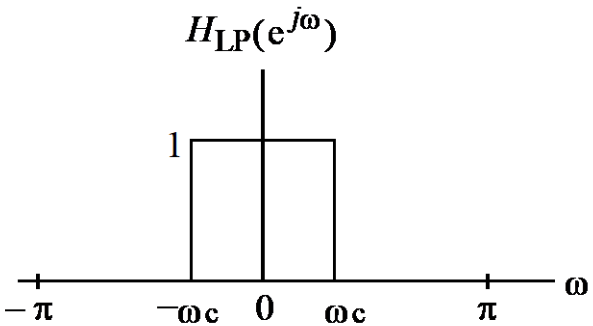
- Note: we discuss only the magnitude approximation problem
- There are four basic types of ideal filters with magnitude responses as shown below



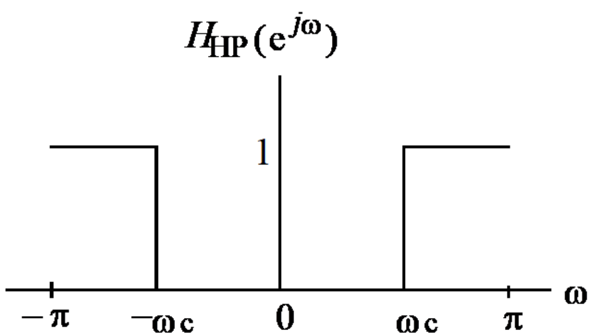
Impulse Responses of Ideal Filters

- Ideal lowpass filter -



$$h_{LP}[n] = \frac{\sin \omega_c n}{\pi n}, \quad -\infty \le n \le \infty$$

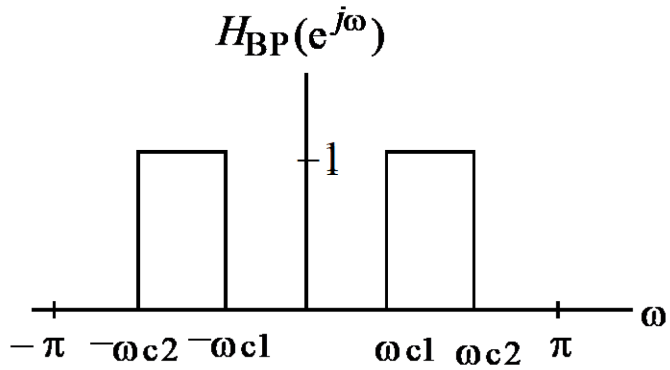
- Ideal highpass filter -



$$h_{HP}[n] = \begin{cases} 1 - \frac{\omega_c}{\pi}, & n = 0 \\ -\frac{\sin(\omega_c n)}{\pi n}, & n \ne 0 \end{cases}$$

Impulse Responses of Ideal Filters

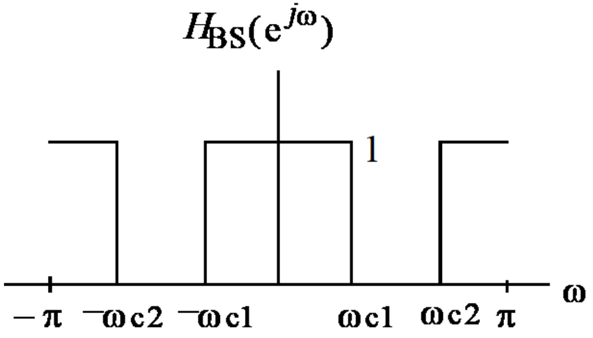
- Ideal bandpass filter -



$$h_{BP}[n] = \begin{cases} \dfrac{\sin(\omega_{c2} n)}{\pi n} - \dfrac{\sin(\omega_{c1} n)}{\pi n}, & n \neq 0 \\ \dfrac{\omega_{c2}}{\pi} - \dfrac{\omega_{c1}}{\pi}, & n = 0 \end{cases}$$

Impulse Responses of Ideal Filters

- Ideal bandstop filter -



$$h_{BS}[n] = \begin{cases} 1 - \dfrac{(\omega_{c2} - \omega_{c1})}{\pi}, & n = 0 \\ \dfrac{\sin(\omega_{c1} n)}{\pi n} - \dfrac{\sin(\omega_{c2} n)}{\pi n}, & n \neq 0 \end{cases}$$

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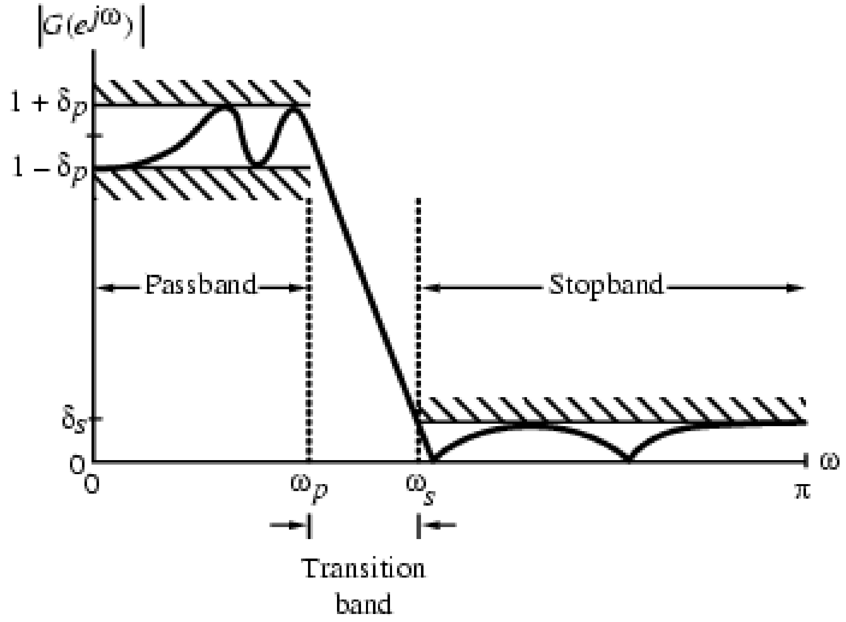
Digital Filter Specifications

- As the impulse response corresponding to each of these ideal filters is noncausal and of infinite length, these filters are not realizable
- In practice, the magnitude response specifications of a digital filter in the passband and in the stopband are given with some acceptable tolerances
- In addition, a transition band is specified between the passband and stopband

Digital Filter Specifications

- For example, the magnitude $\left|G(e^{j\omega})\right|$ response of a digital lowpass filter may be given as indicated below



Digital Filter Specifications

- As indicated in the figure, in the **passband**, defined by $0 \le \omega \le \omega_p$, we require that $|G(e^{j\omega})| \cong 1$ with an error $\pm\delta_p$, i.e.,

$$1-\delta_p \le |G(e^{j\omega})| \le 1+\delta_p, \quad |\omega| \le \omega_p$$

- In the **stopband**, defined by $\omega_s \le \omega \le \pi$, we require that $|G(e^{j\omega})| \cong 0$ with an error δ_s, i.e.,

$$|G(e^{j\omega})| \le \delta_s, \quad \omega_s \le |\omega| \le \pi$$

Digital Filter Specifications

- ω_p - **passband edge frequency**
- ω_s - **stopband edge frequency**
- δ_p - **peak ripple value** in the passband
- δ_s - **peak ripple value** in the stopband
- Since $G(e^{j\omega})$ is a periodic function of ω, and $|G(e^{j\omega})|$ of a real-coefficient digital filter is an even function of ω
- As a result, filter specifications are given only for the frequency range $0 \leq \omega \leq \pi$

Digital Filter Specifications

- Specifications are often given in terms of **loss function** $\mathcal{A}(\omega) = -20\log_{10}\left|G(e^{j\omega})\right|$ in dB
- **Peak passband ripple**

$$\alpha_p = -20\log_{10}(1-\delta_p) \text{ dB}$$

- **Minimum stopband attenuation**

$$\alpha_s = -20\log_{10}(\delta_s) \text{ dB}$$

Digital Filter Specifications

- For the normalized specification, maximum value of the gain function or the minimum value of the loss function is 0 dB
- **Maximum passband attenuation** -

$$\alpha_{max} = 20\log_{10}\left(\sqrt{1+\varepsilon^2}\right) \quad \text{dB}$$

- For $\delta_p << 1$, it can be shown that

$$\alpha_{max} \cong -20\log_{10}(1-2\delta_p) \text{ dB}$$

Digital Filter Specifications

- In practice, passband edge frequency F_p and stopband edge frequency F_s are specified in Hz
- For digital filter design, normalized bandedge frequencies need to be computed from specifications in Hz using

$$\omega_p = \frac{\Omega_p}{F_T} = \frac{2\pi F_p}{F_T} = 2\pi F_p T$$

$$\omega_s = \frac{\Omega_s}{F_T} = \frac{2\pi F_s}{F_T} = 2\pi F_s T$$

Digital Filter Specifications

- Example - Let $F_p = 7$ kHz, $F_s = 3$ kHz, and $F_T = 25$ kHz
- Then

$$\omega_p = \frac{2\pi(7\times 10^3)}{25\times 10^3} = 0.56\pi$$

$$\omega_s = \frac{2\pi(3\times 10^3)}{25\times 10^3} = 0.24\pi$$

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Selection of Filter Type

- The transfer function $H(z)$ meeting the frequency response specifications should be a causal transfer function
- For IIR digital filter design, the IIR transfer function is a real rational function of z^{-1} :

$$H(z) = \frac{p_0 + p_1 z^{-1} + p_2 z^{-2} + \cdots + p_M z^{-M}}{d_0 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_N z^{-N}}, \quad M \leq N$$

- $H(z)$ must be a stable transfer function and must be of lowest order N for reduced computational complexity

Selection of Filter Type

- For FIR digital filter design, the FIR transfer function is a polynomial in z^{-1} with real coefficients:

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n}$$

- For reduced computational complexity, degree N of $H(z)$ must be as small as possible
- If a linear phase is desired, the filter coefficients must satisfy the constraint:

$$h[n] = \pm h[N-n]$$

Selection of Filter Type

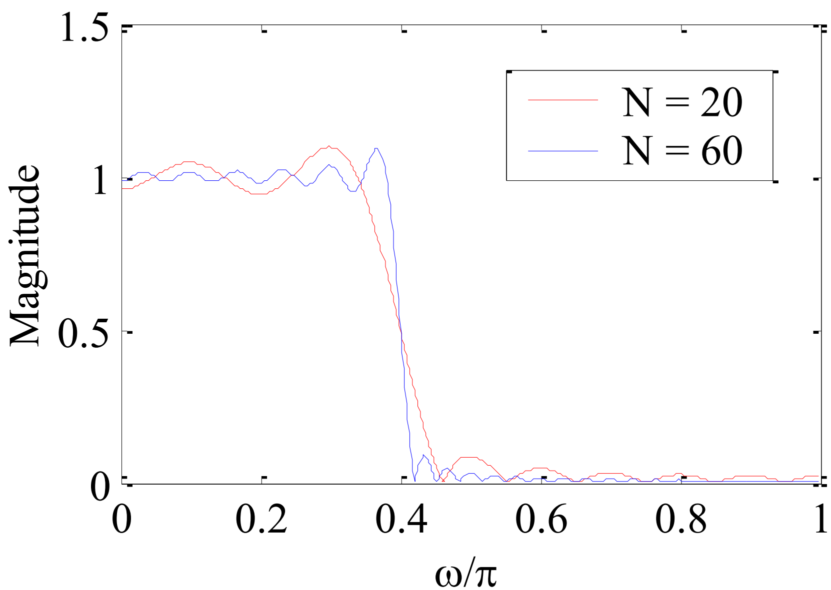
- Advantages in using an FIR filter -

  (1) Can be designed with exact linear phase,

  (2) Filter structure always stable with quantized coefficients
- Disadvantages in using an FIR filter - Order of an FIR filter, in most cases, is considerably higher than the order of an equivalent IIR filter meeting the same specifications, and FIR filter has thus higher computational complexity

Gibbs Phenomenon

- Gibbs phenomenon - Oscillatory behavior in the magnitude responses of causal FIR filters obtained by truncating the impulse response coefficients of ideal filters



Gibbs Phenomenon

- As can be seen, as the length of the lowpass filter is increased, the number of ripples in both passband and stopband increases, with a corresponding decrease in the ripple widths
- Height of the largest ripples remain the same independent of length
- Similar oscillatory behavior observed in the magnitude responses of the truncated versions of other types of ideal filters

Gibbs Phenomenon

- Gibbs phenomenon can be reduced either:

  (1) Using window that tapers smoothly to zero at each end

  (2) Providing a smooth transition from passband to stopband in the magnitude specifications

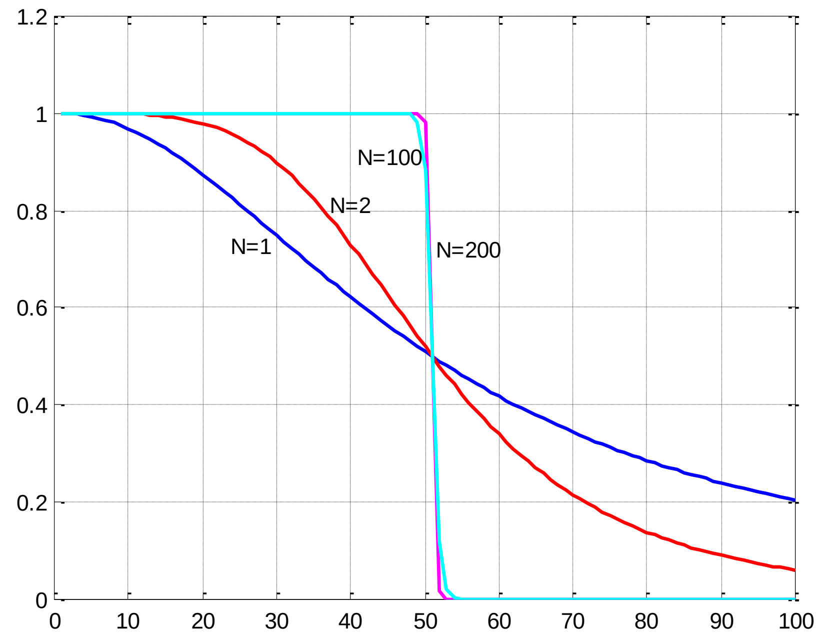
Butterworth Lowpass Filter

- This filter is characterized by the property that its magnitude response is *flat* in both passband and stopband.

The magnitude-squared response of an *N*-order lowpass filter

Ω_c is the cutoff frequency in rad/sec.

$$|H_a(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$



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Chebyshev Lowpass Filter

- Chebyshev-I filters
  - Have *equiripple* response in the *passband*
- Chebyshev-II filters
  - Have *equiripple* response in *stopband*
- Butterworth filters
  - Have *monotonic* response in both bands
- We note that by choosing a filter that has an equripple rather than a monotonic behavior, we can obtain a *low-order* filter.
- Therefore Chebyshev filters provide *lower order* than Buttworth filters for the same specifications.

Elliptic Lowpass Filters

- These filters exhibit equiripple behavior in the passband as well as the stopband. They are similar in magnitude response characteristics to the FIR equiripple filters.
- Therefore elliptic filters are optimum filters in that they achieve the minimum order N for the given specifications
- These filters, for obvious reasons, are very difficult to analyze and therefore, to design.
- It is not possible to design them using simple tools, and often programs or tables are needed to design them.

References

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