Jitter and Phase Noise

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Introduction

Phase Noise

Jitter

Jitter Measurement

Conclusions

Presentation Outline

- Introduction
- Phase Noise
- Jitter
- Jitter Measurement
- Conclusions

Introduction

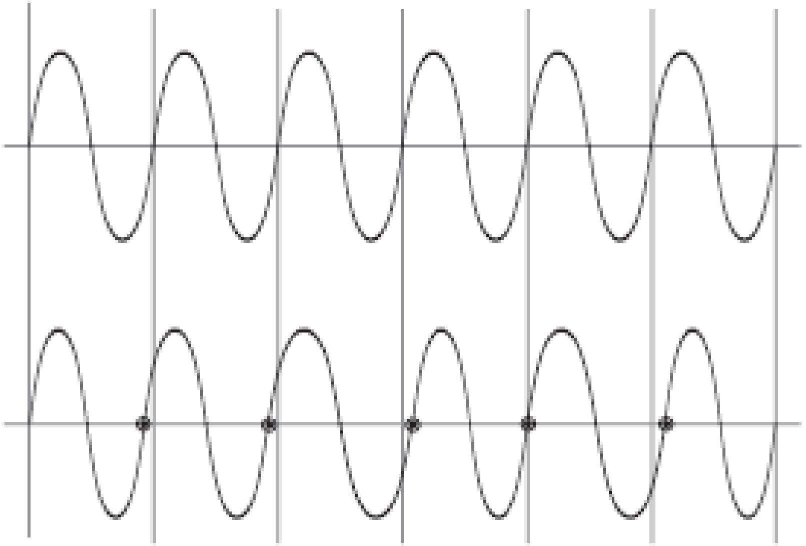
- Difference between phase noise and jitter
- Origin of phase noise
- Relation between phase noise and jitter
- Jitter measurement in data acquisition systems

Phase Noise

Outline

- What is Phase Noise
- Why it is important
- Where does it come from
- How do we specify it

What is Phase Noise



$$v(t) = A \cdot \cos\left[2\pi \cdot f_c \cdot t + \varphi(t)\right]$$

- φ accounts for deviations in phase and in frequency from its nominal value of f_c

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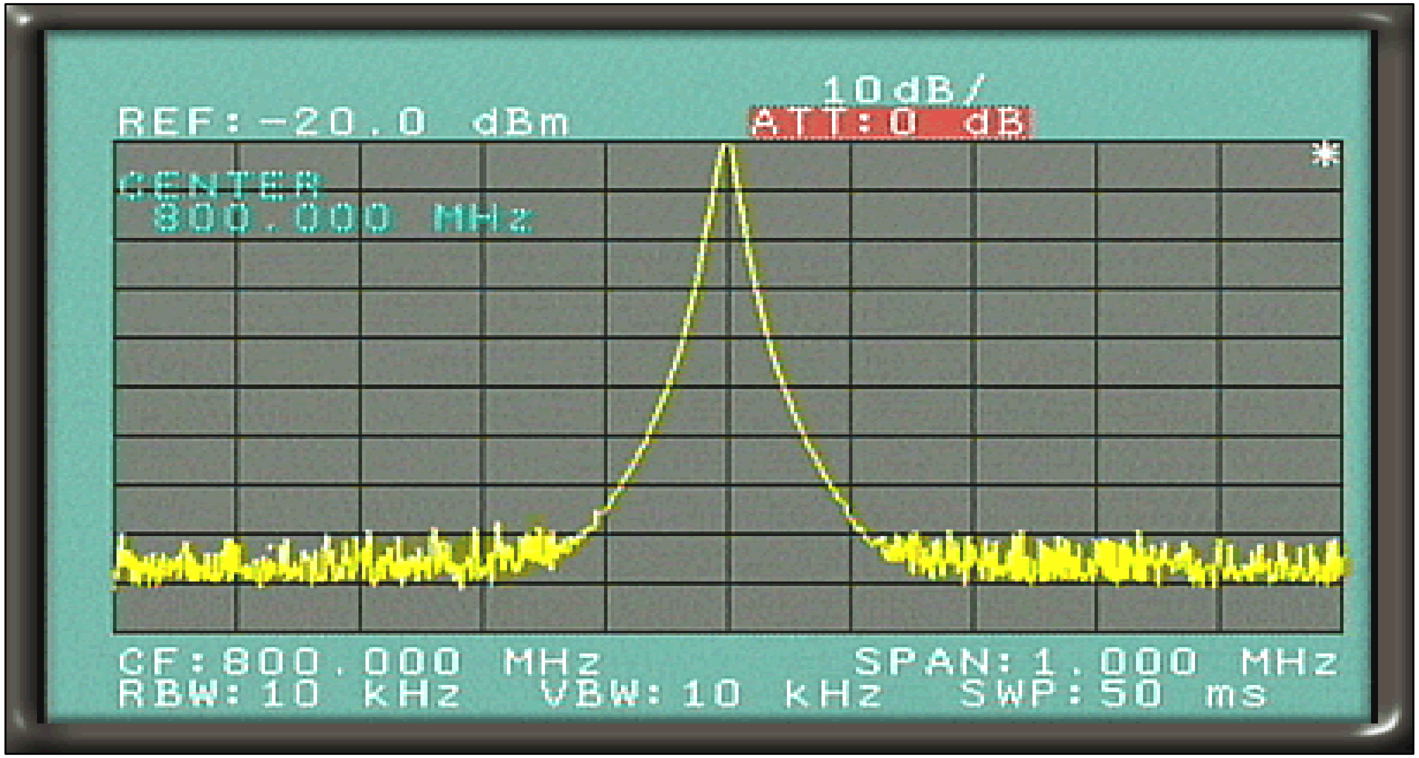
PN in the Frequency Domain



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Power Spectral Density

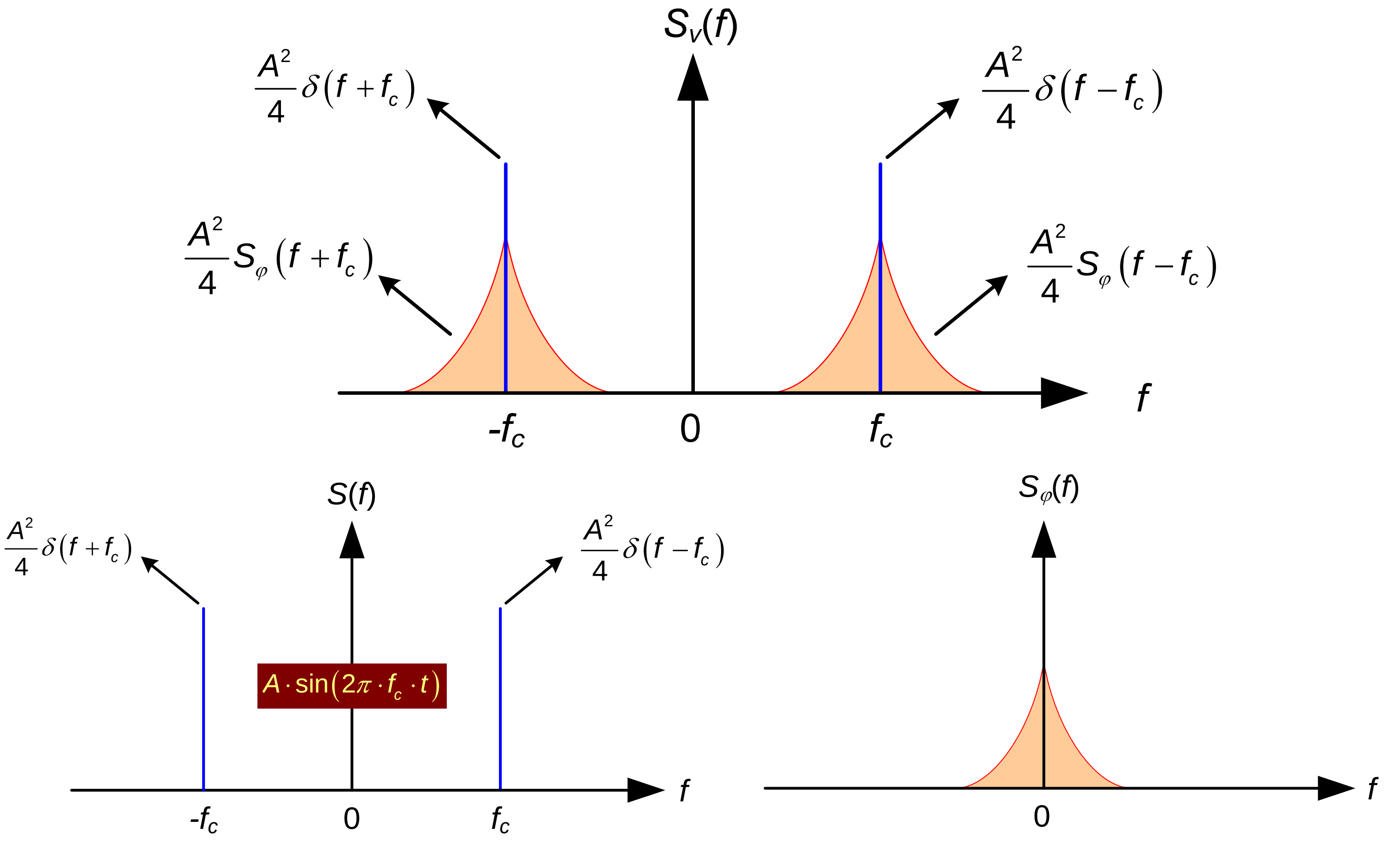
$$v(t) = A \cdot \sin\left[2\pi \cdot f_c \cdot t + \varphi(t)\right]$$

$$v(t) \approx A \cdot \sin\left(2\pi \cdot f_c \cdot t\right) + A \cdot \varphi(t) \cdot \cos\left(2\pi \cdot f_c \cdot t\right)$$

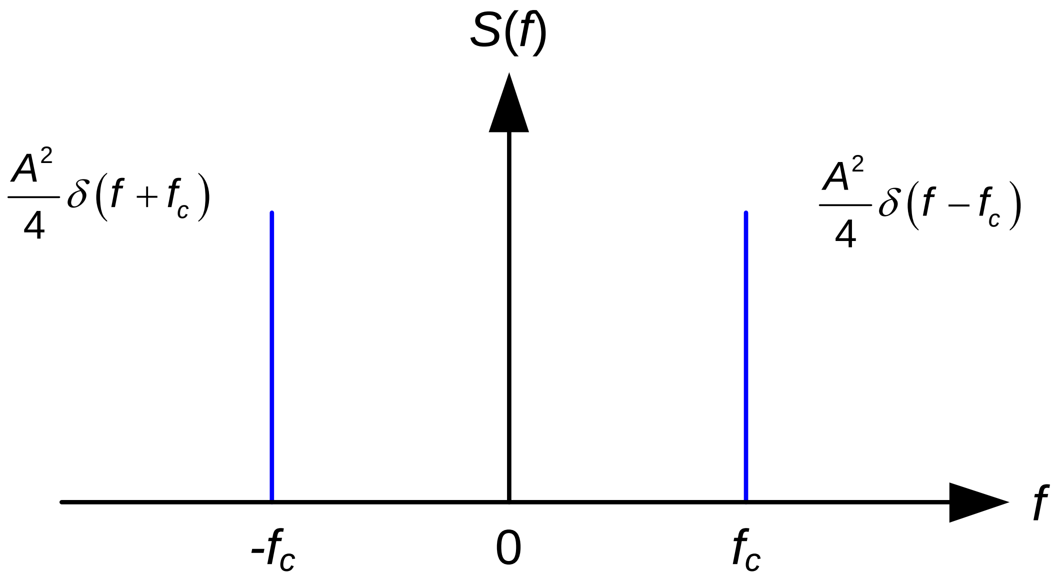
$$S_v(f) = \frac{1}{T}\left|\mathcal{F}\left\{v(t)\right\}\right|^2 \qquad S_\varphi(f) = \frac{1}{T}\left|\mathcal{F}\left\{\varphi(t)\right\}\right|^2$$

$$S_v(f) = \frac{A^2}{4}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A^2}{4}\left[S_\varphi(f - f_c) + S_\varphi(f + f_c)\right]$$

Power Spectral Density



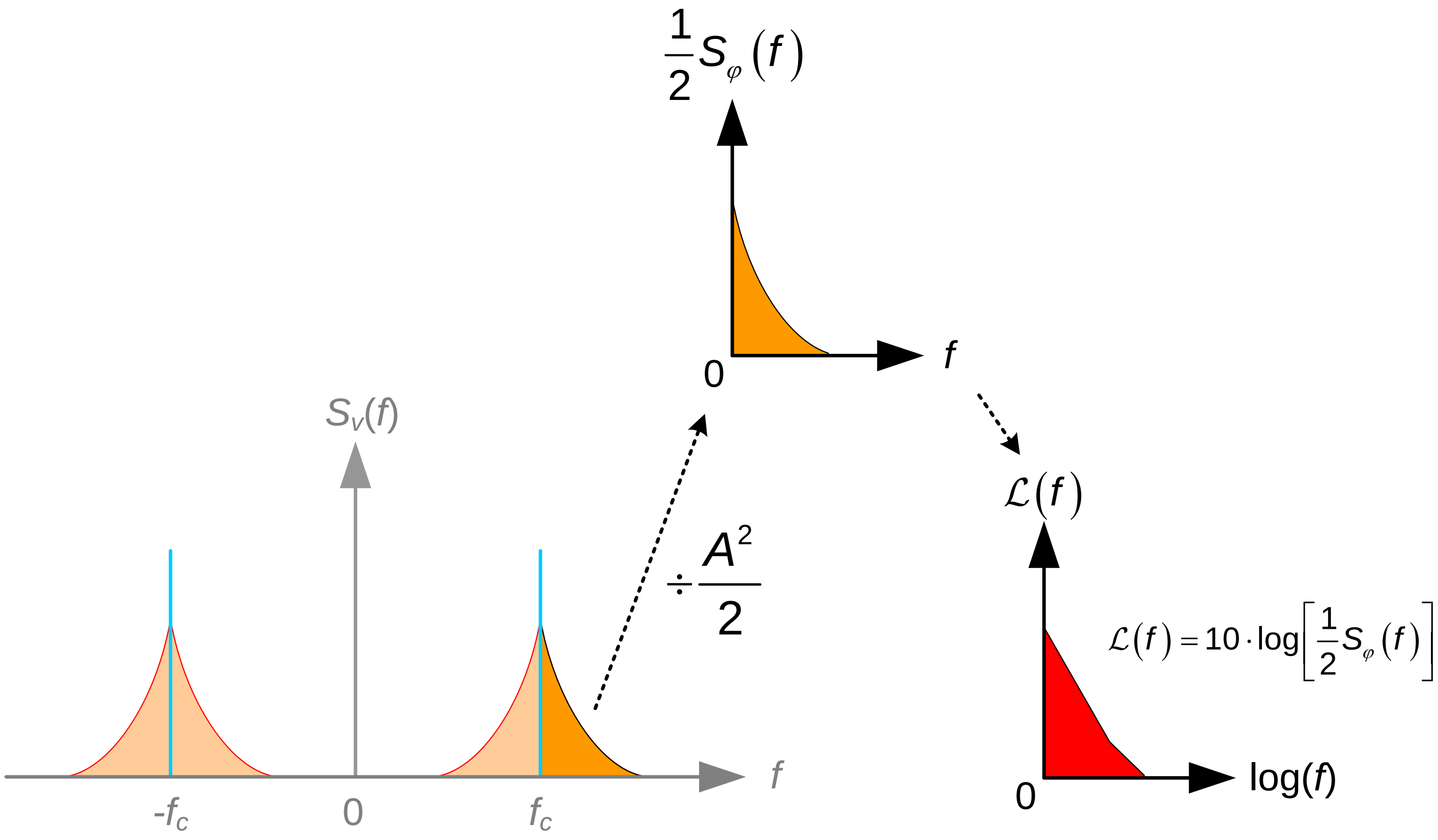
Sine wave Power



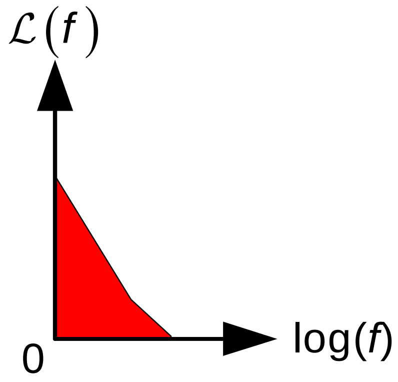
$$P = \frac{1}{T}\int_0^T v^2(t)\cdot dt \qquad P = \int_{-\infty}^{\infty} S(f)\cdot df$$

$$\boxed{P = \frac{A^2}{2}}$$

Phase Noise Spectrum



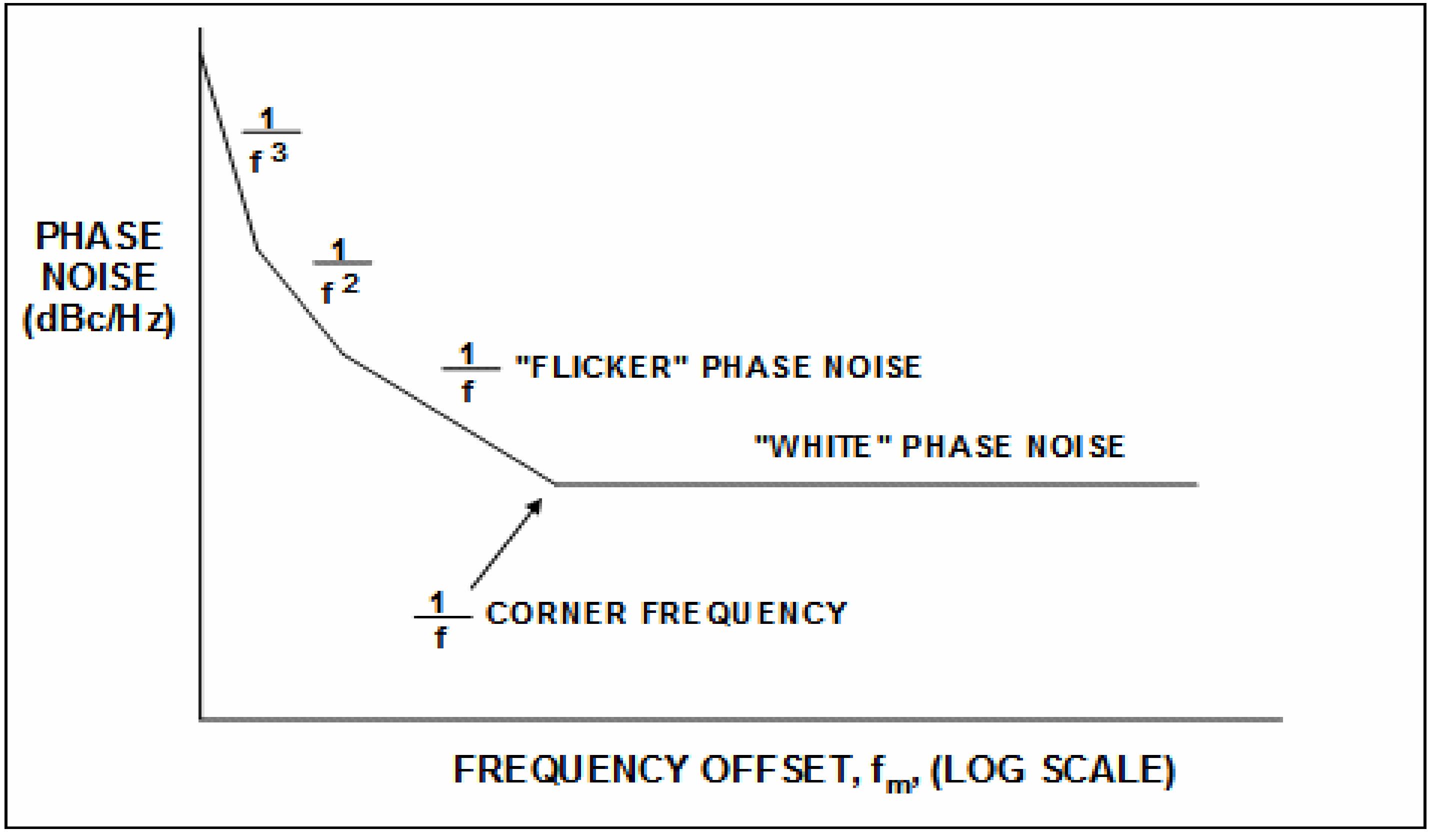
Phase Noise Spectrum Units



$$\mathcal{L}(f) = 10 \cdot \log\left[\frac{1}{2} S_\varphi(f)\right]$$

- S_φ is expressed in rad²/Hz
- $\mathcal{L}$ is expressed in dBc/Hz
  - Note that **Hz** in this case refers to the argument of the logarithm

Typical Phase Noise Spectrum



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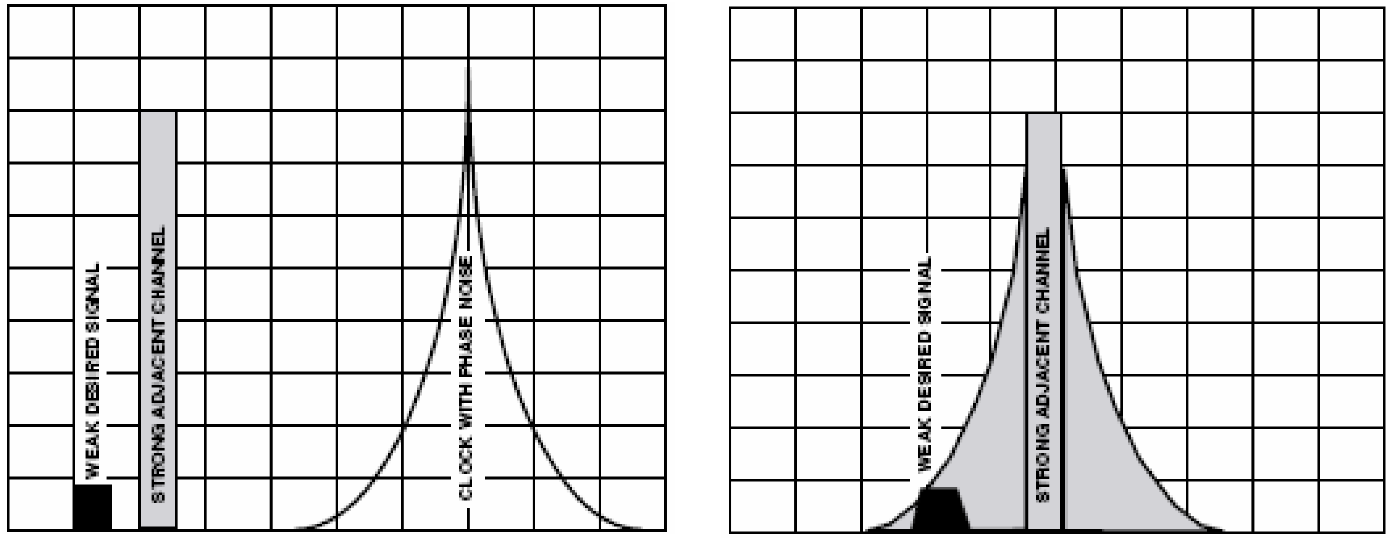
Why it is Important



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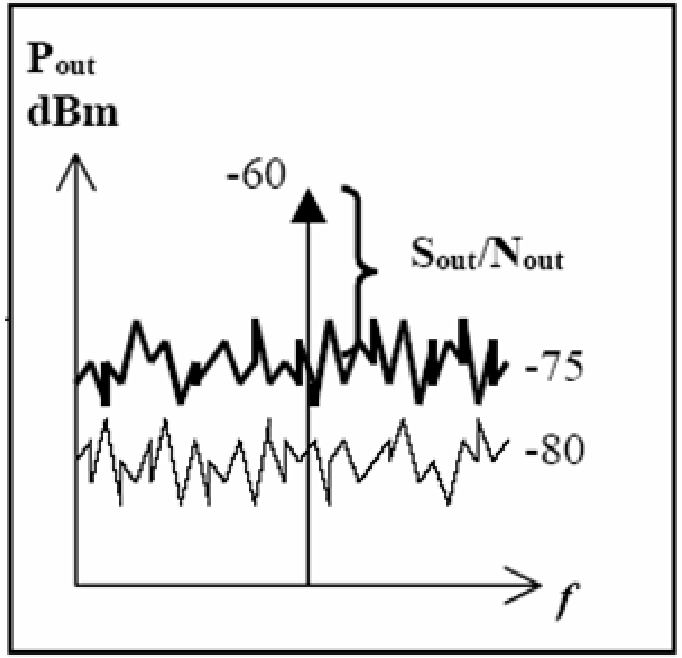
- The presence of Phase Noise can mask important signals.

Why it is Important



- Increase in Phase Noise degrades SNR

Where Does it Come From?

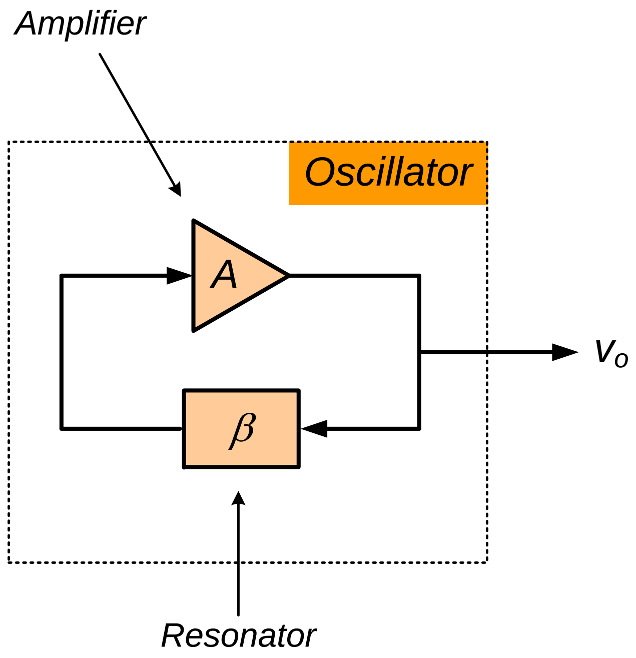
- Different Types of Oscillators
  - Resonator based (feedback)
  - Relaxation
  - Ring Oscillators

Phase Noise

Resonator Based Oscillator

Resonator Based Oscillator

- Feedback loop
- Active element *A*
- Resonator β



Barkhausen Condition

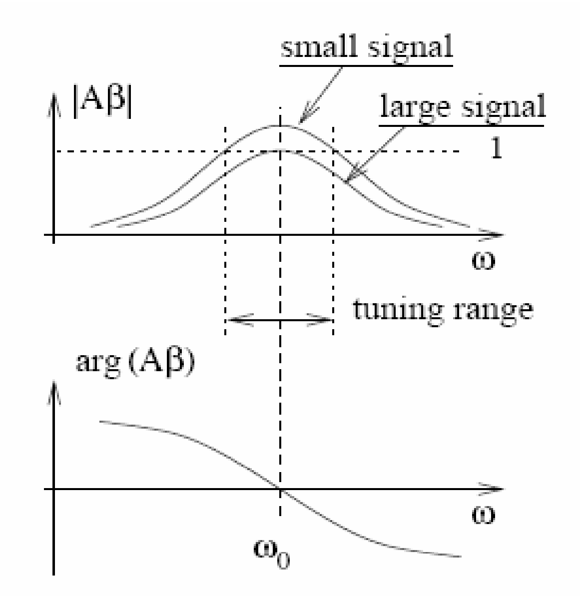
$$\left|A \cdot \beta\left(\omega_o\right)\right| = 1$$

$$A \cdot \beta\left(\omega_o\right) = 1$$

$$\arg\left[A \cdot \beta\left(\omega_o\right)\right] = 0$$

Oscillations

- Noise or switch-on transient
- Energy on ω_0 will rise exponential because $|A\beta| > 1$
- Amplitude is stabilized by automatic gain control or amplifier saturation



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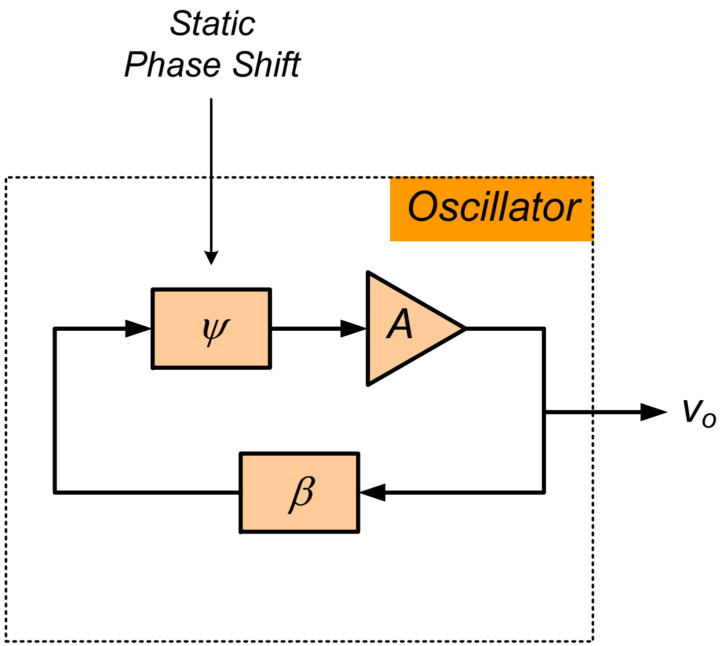
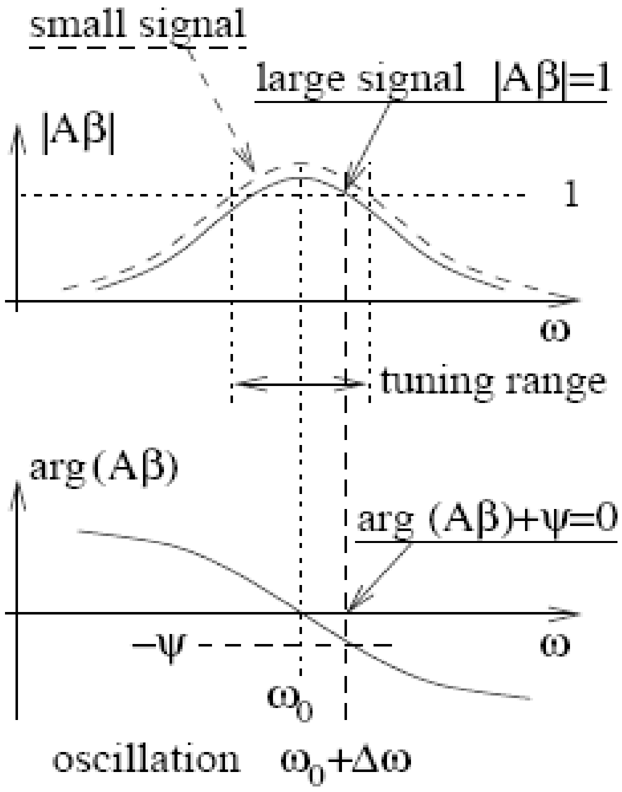
Tuning the Oscillator

- Static phase shift
  - Used mainly in microwave oscillators
- Change the natural frequency of the resonator
  - Used mainly in crystal oscillators

Static Phase Shift





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Static Phase Shift

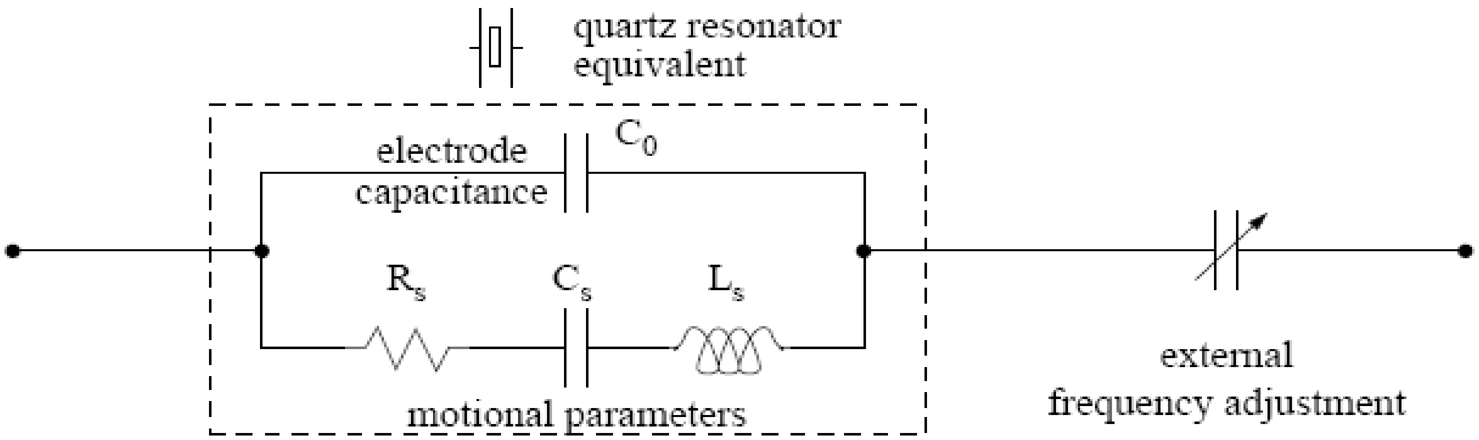
$$\arg\left[A\cdot\beta(\omega)\right]+\psi=0 \quad \text{at} \quad \omega=\omega_o+\Delta\omega$$

$$\Delta\omega\cdot\frac{d}{d\omega}\arg\left[A\cdot\beta(\omega)\right]\approx-\psi$$

$$\frac{d}{d\omega}\arg\left[A\cdot\beta(\omega)\right]\approx-\frac{2Q}{\omega_o}$$ for a 2nd order resonator with large Q

$$\frac{\Delta\omega}{\omega_o}=\frac{\psi}{2Q} \quad \text{for} \quad \frac{\Delta\omega}{\omega_o}=\frac{1}{2Q}$$

Change Natural Frequency of the Resonator



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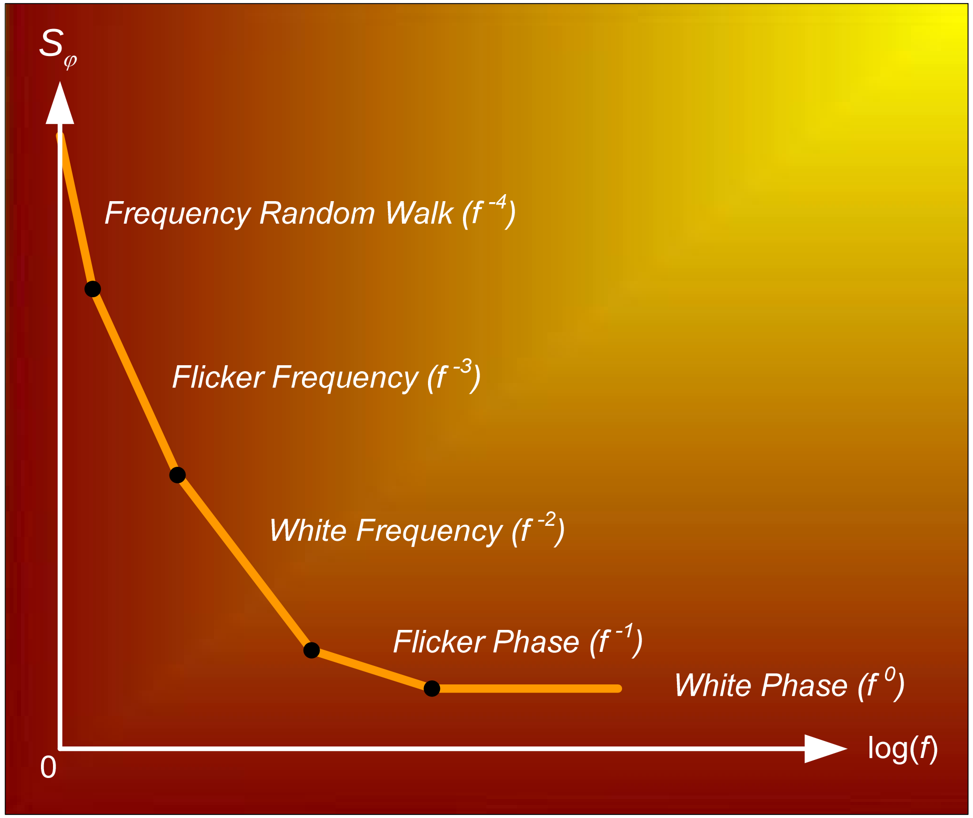
Different Kinds of Phenomena



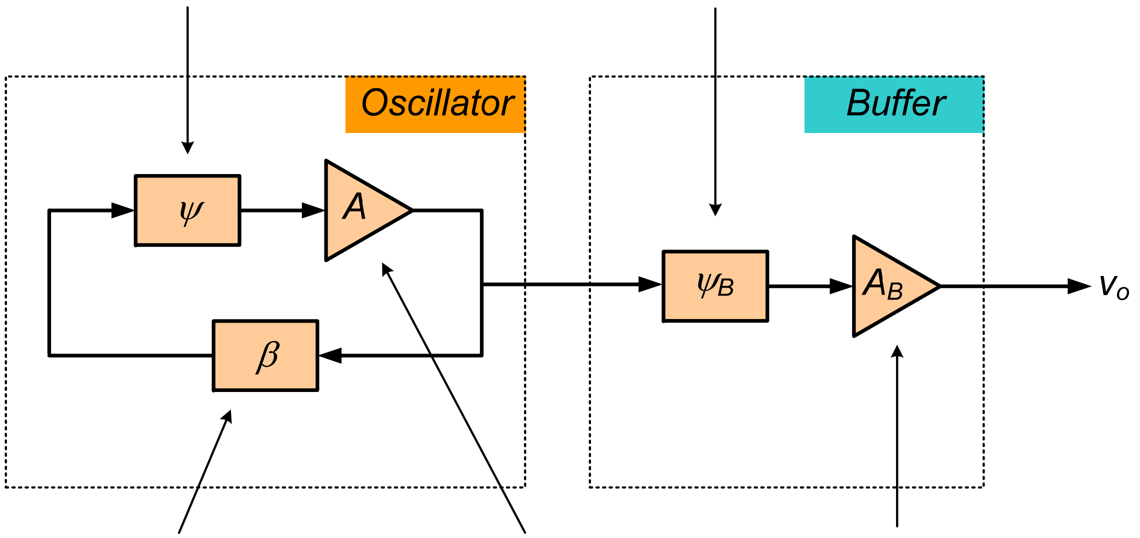
Analysis of the Oscillator

Random Phase

① *Slow Fluctuations*

② *Fast Fluctuations*

Random Phase



Resonator

⑤ *Additive Noise*

⑥ *Flicker Frequency*

⑦ *Frequency Random Walk*

Amplifier

③ *Additive Noise*

④ *Flicker Noise*

Buffer Amplifier

⑧ *Additive Noise*

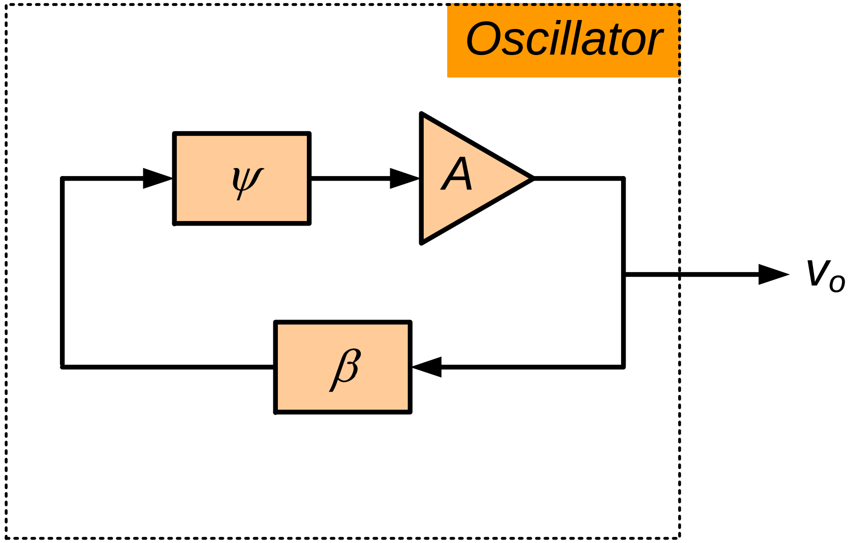
⑨ *Flicker Noise*

Analysis of the Oscillator

- Phase Noise of Amplifier and other components (*Leeson Effect*)
  - Slow Fluctuations
  - Fast Fluctuations
- Amplifier Noise
  - Additive White Noise
  - Flicker Noise
- Resonator Noise
  - Additive White Noise
  - Flicker Frequency
  - Frequency Random Walk
- Buffer Amplifier Noise
  - Additive White Noise
  - Flicker Noise

Leeson Formula

- Consider an ideal resonator
  - No frequency fluctuations
  - Large quality factor Q
- Account all phase noise sources by ψ



Quasi-static Perturbation

- The slow components of φ, that is, slower than the relaxation time τ, can be treated as stationary

$$\tau = \frac{2Q}{\omega_o} \rightarrow \text{relaxation time}$$

$$\Delta f = \frac{f_o}{2Q}\varphi(t)$$

Spectral Density of Frequency Deviations

$$\Delta f = \frac{f_o}{2Q}\varphi(t)$$

$$S_{\Delta f}(f) = \left(\frac{f_o}{2Q}\right)^2 S_{\varphi}(f)$$

Spectral Density of Phase Noise

$$\varphi(t) = 2\pi \int \Delta f(t) \cdot dt$$

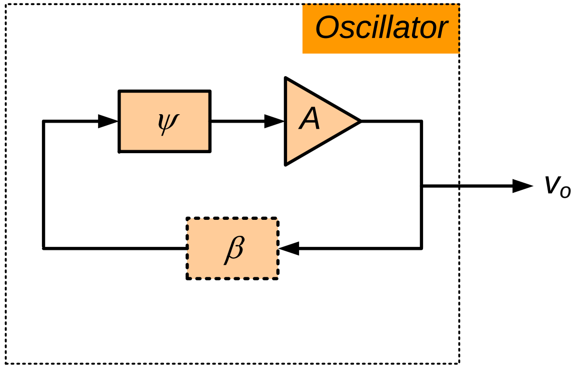
Integration in the time domain corresponds to division by $j\omega$ in the frequency domain

Division by $j\omega$ in the frequency domain corresponds to division by $(j\omega)^2$ in the spectrum

$$S_\varphi(f) = \frac{1}{f^2}\left(\frac{f_o}{2Q}\right)^2 S_\psi(f)$$

Fast Fluctuations

- The fast components of φ , that is, faster than the relaxation time τ, the resonator can be treated as an open circuit
- The output noise is not fed back



$$S_{\varphi}(f) = S_{\psi}(f)$$

Slow and Fast Fluctuations

Slow Fluctuations → $S_\varphi(f) = \frac{1}{f^2}\left(\frac{f_o}{2Q}\right)^2 S_\psi(f)$

Fast Fluctuations → $S_\varphi(f) = S_\psi(f)$

$$S_\varphi(f) = \left[1 + \frac{1}{f^2}\left(\frac{f_o}{2Q}\right)^2\right] S_\psi(f)$$

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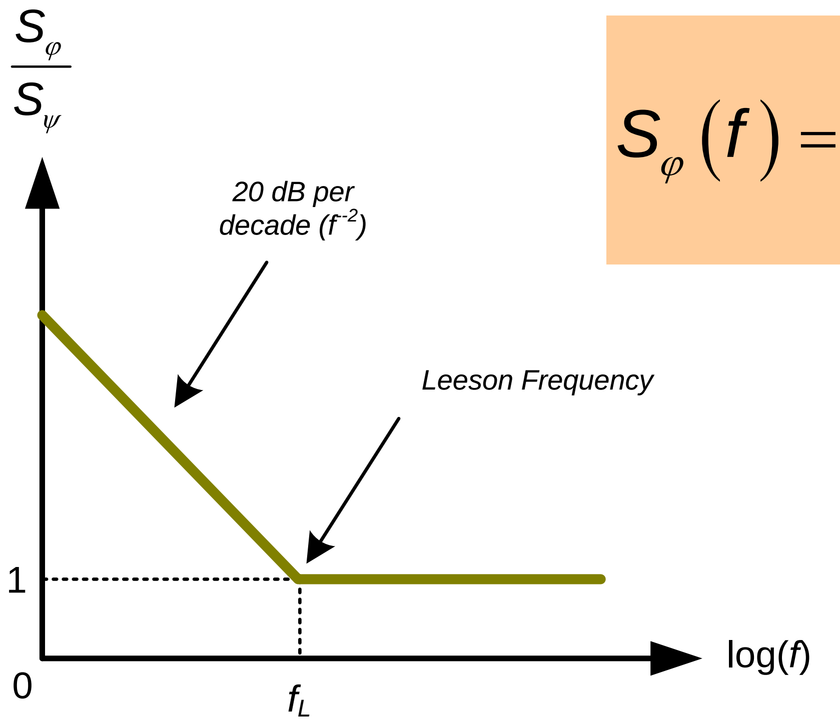
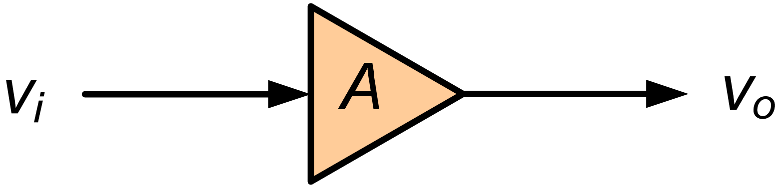
Leeson Formula



$$S_\varphi(f) = \left[1 + \left(\frac{f_L}{f}\right)^2\right] S_\psi(f)$$

$$f_L = \frac{f_o}{2Q}$$

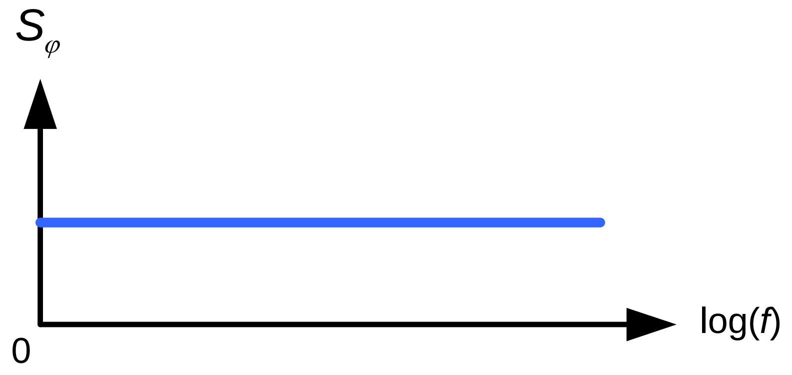
Amplifier Analysis



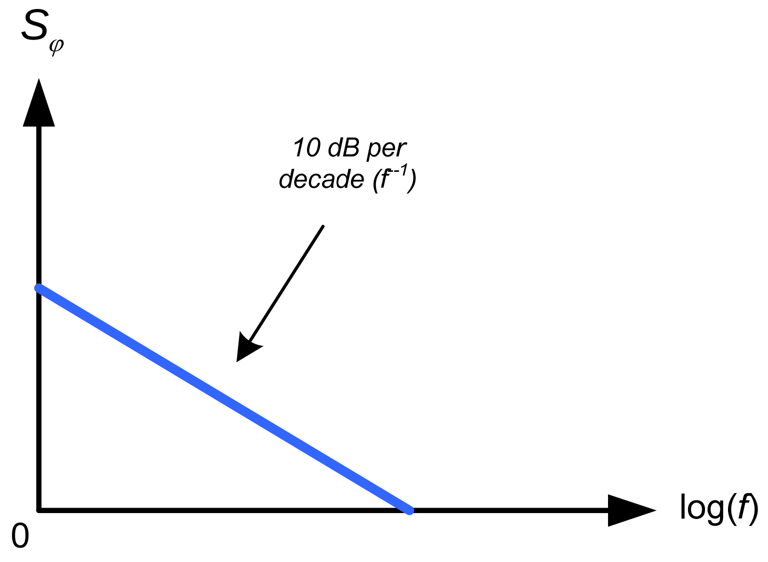
Additive White Noise in the Amplifier

- White Noise has a constant Power Density
- It depends on the
  - Noise Figure F
  - Carrier power P_0
  - Ambient temperature T_0 (290 K)
  - Boltzman Constant k ($1.3806503 \times 10^{-23}$ J/K)

$$S_\varphi(f) = \frac{FkT_0}{P_0}$$



Flicker Noise in the Amplifier



$$S_\varphi(f) \propto \frac{1}{f}$$

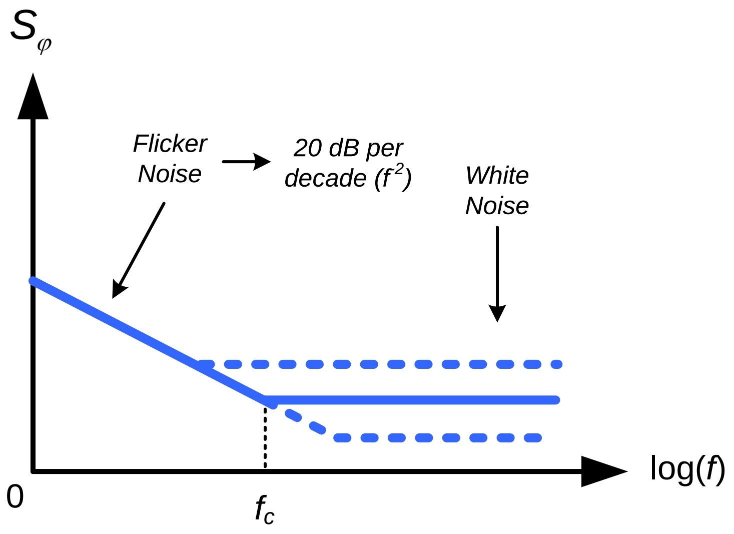
Unconverted Flicker Noise

$$x(t) = V_o \cos(\omega_0 t) + n(t) \qquad f(x) = a_0 + a_1 x + a_2 x^2$$

$$y(t) = f(x(t)) = a_0 + a_1 V_0 \cos(\omega_0 t) + a_1 n(t) +$$
$$+ a_2 V_0^2 \cos^2(\omega_0 t) + a_2 n(t) + a_2 V_0 n(t) \cos(\omega_0 t)$$

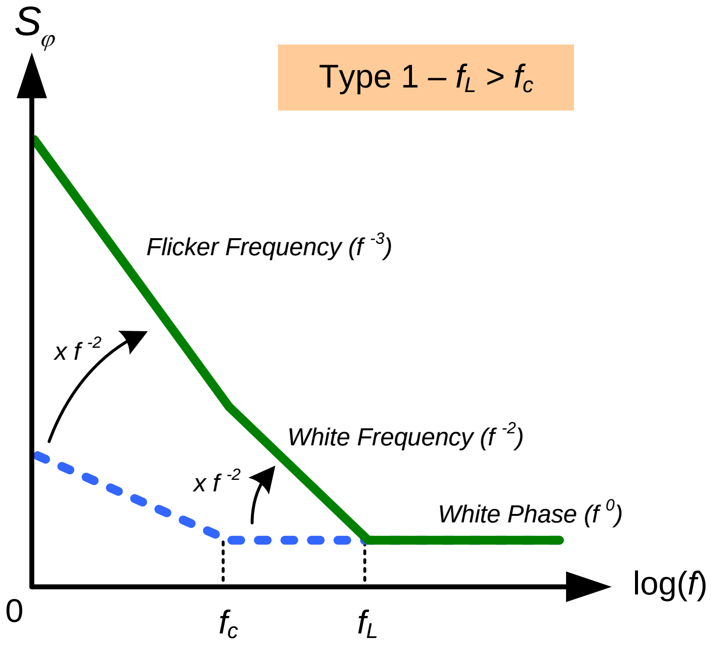
Phase Noise of the Amplifier

- White Noise and Flicker Noise are independent
- White Noise is inversely proportional to carrier power
- Flicker Noise is independent of carrier power



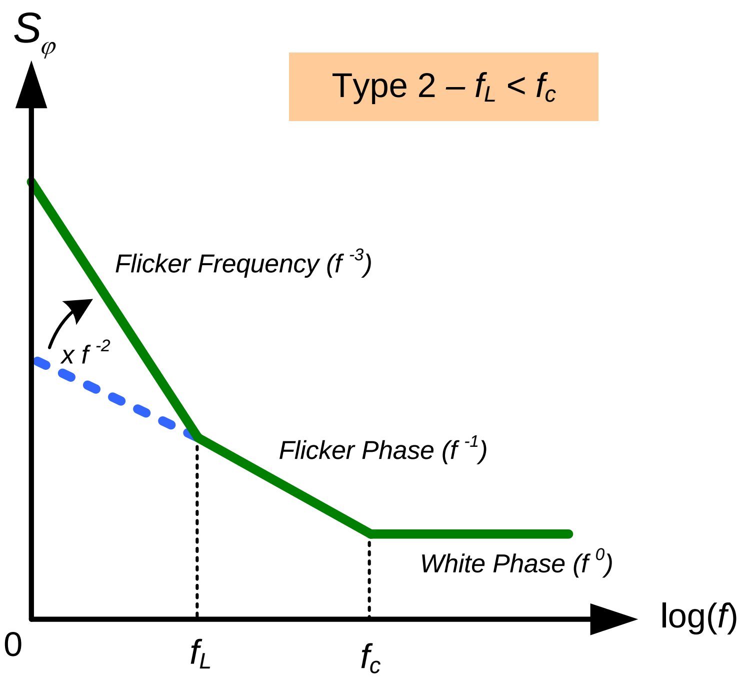
Amplifier + Leeson (Type 1)

- Typical of
  - Microwave oscillators
  - High frequency piezoelectric oscillators (>100 MHz)
- f_L is high because
  - High resonant frequency
  - Low Q



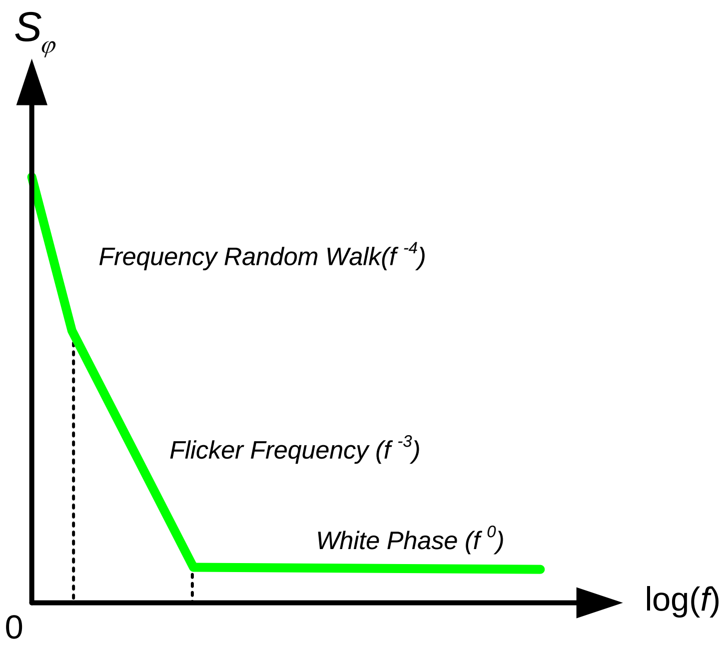
Amplifier + Leeson (Type 2)

- Low frequency (5 to 10 MHz), high stability quartz oscillators
- f_L is low due to high Q (> 10^6)

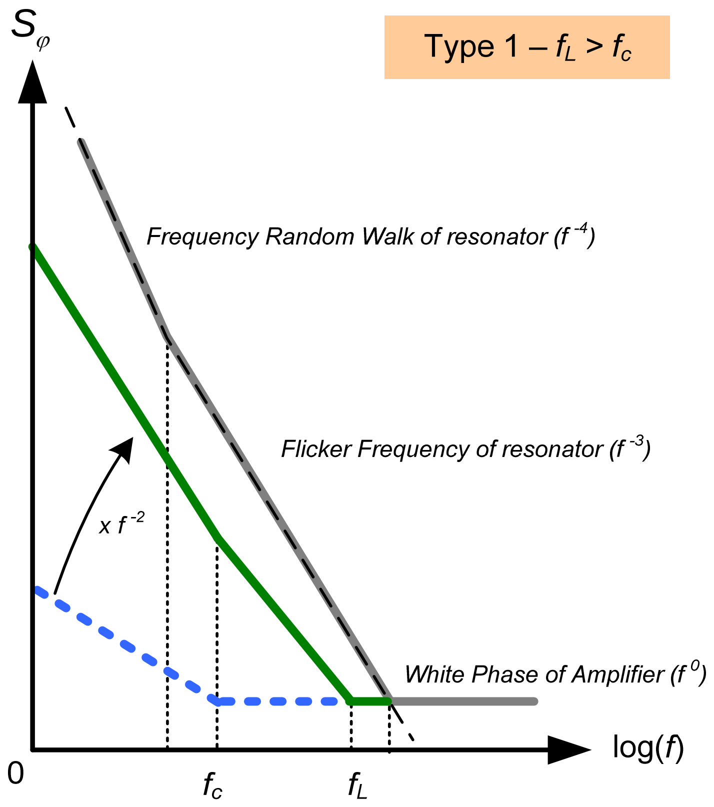


Resonator Noise

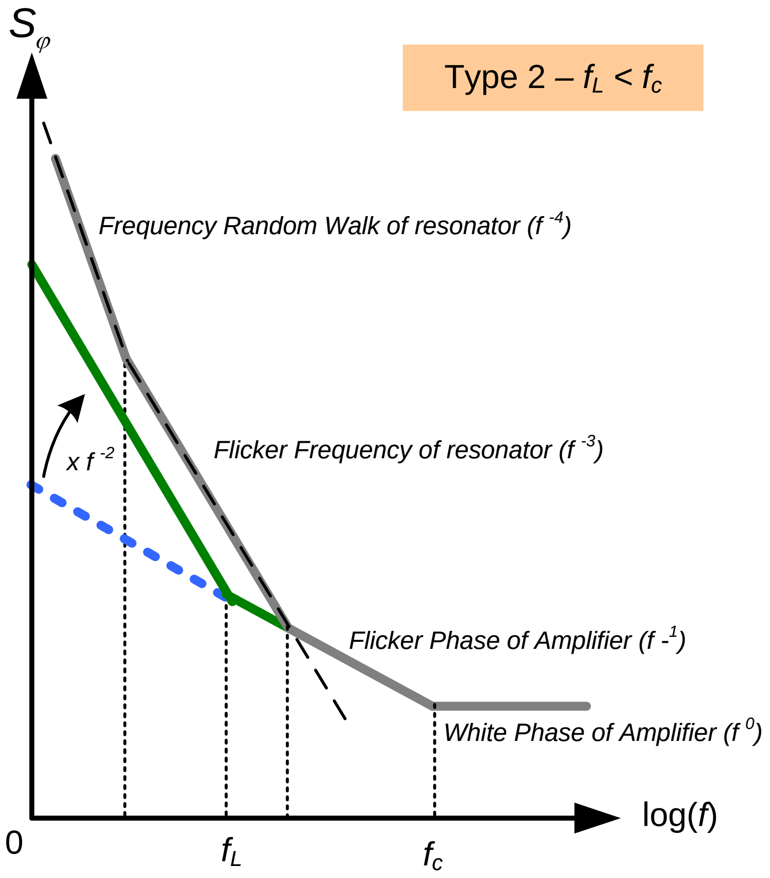
- Types of noise present
  - White noise (not significant)
  - Flicker frequency
  - Frequency Random Walk



Effect of Resonator Noise (Type 1)



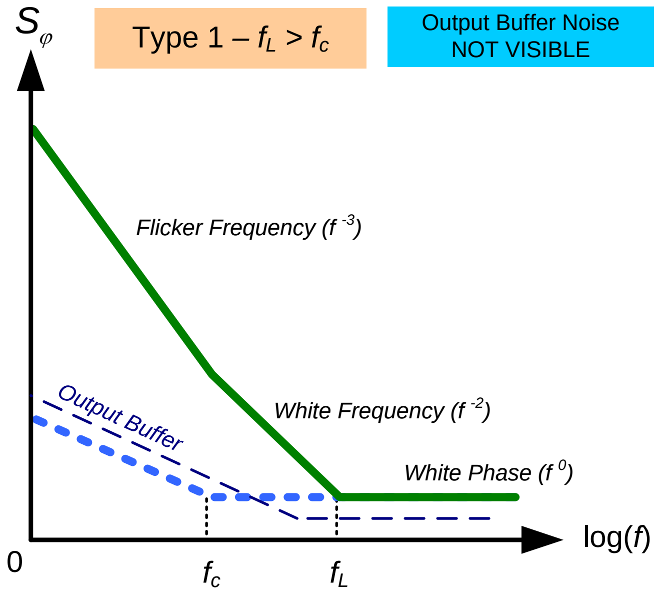
Effect of Resonator Noise (Type 2)

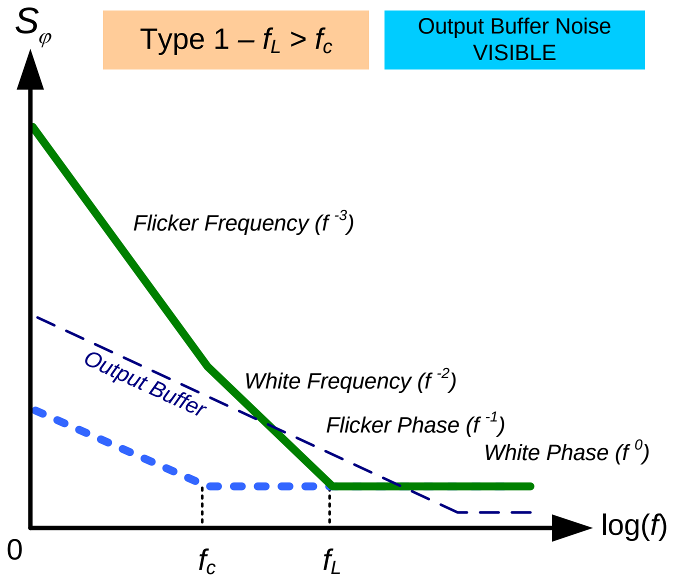


Buffer Amplifier

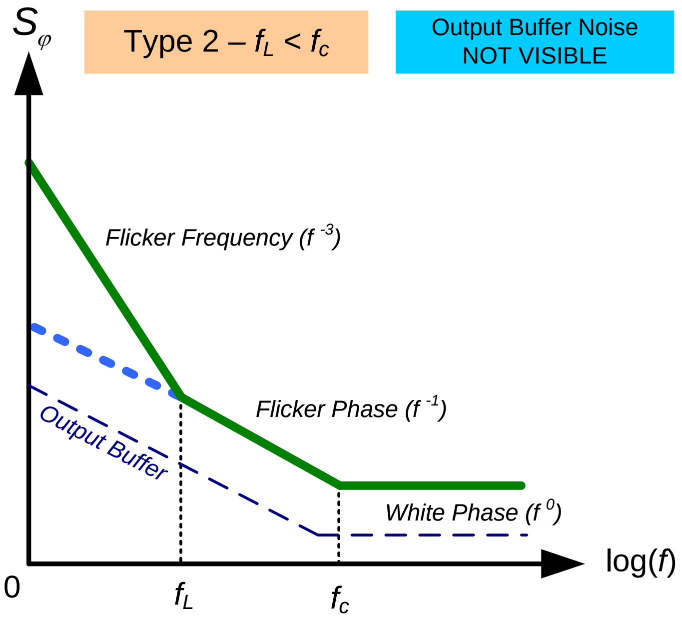
- White Noise
- Flicker Noise

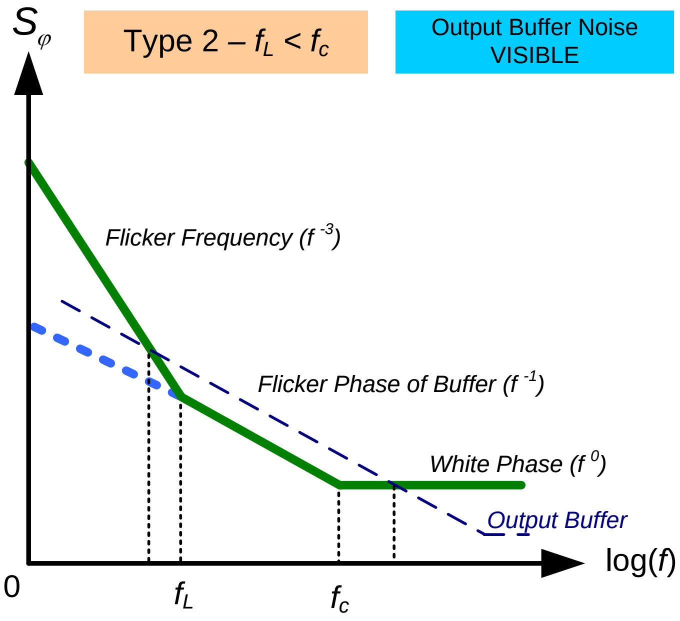
Effect of Buffer Amplifier Noise on Type 1





Effect of Buffer Amplifier Noise on Type 2

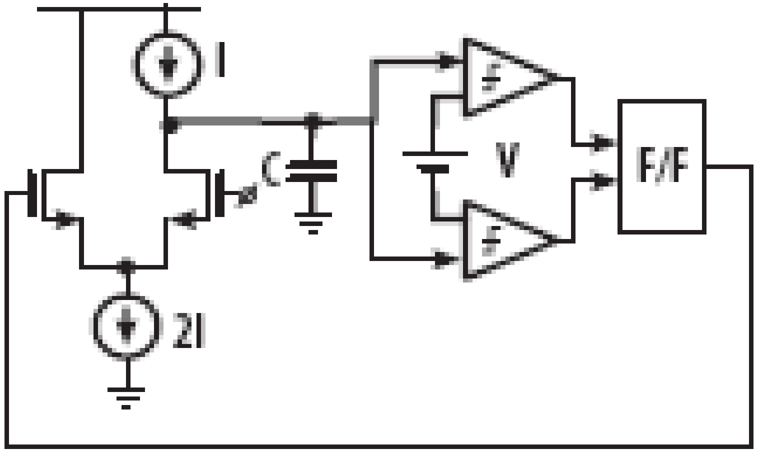


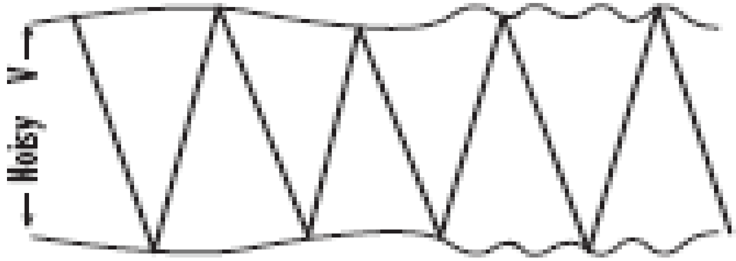


Phase Noise

Relaxation Oscillator

Relaxation Oscillator





Oscillation Frequency

- Frequency Modulation due to:
  - Noise in the charging current
  - Noise on the reference voltages

$$f_c = \frac{I}{2C\Delta V_t}$$

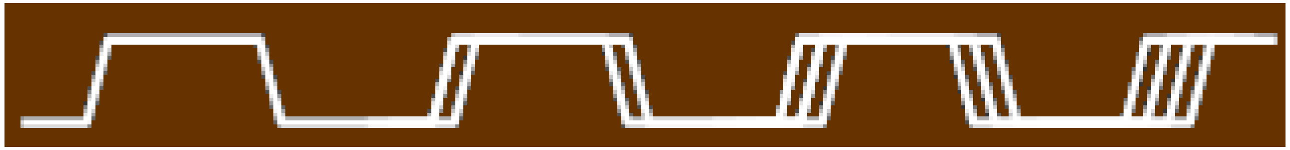
Phase Noise Spectrum

$$S_{\varphi}(f) = \frac{1}{I^2}\left(\frac{f_c}{f}\right)^2 S_{i_n}(f) \qquad S_{\varphi}(f) = \frac{1}{(\Delta v_t)^2}\left(\frac{f_c}{f}\right)^2 S_{v_n}(f)$$

- Flicker noise in I or Δv_t will appear as f^{-3} phase noise

Jitter

What is Jitter



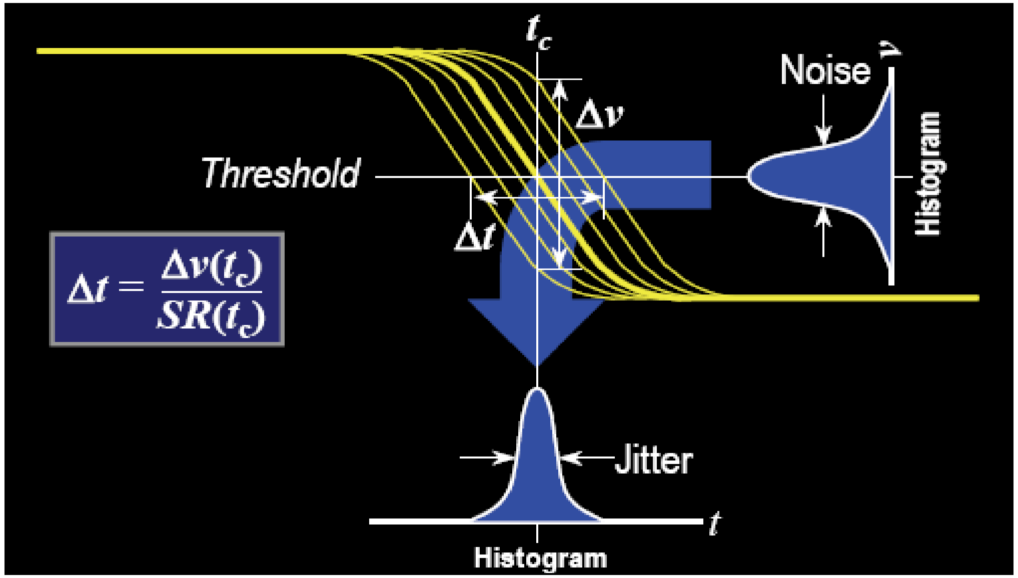
$$v(t) = A \cdot \cos\left[2\pi \cdot f_c \cdot t + \varphi(t)\right]$$

$$v(t) = A \cdot \cos\left[2\pi \cdot f_c \cdot \left(t + j(t)\right)\right]$$

$$j(t) = \frac{\varphi(t)}{2\pi \cdot f_c}$$

What Causes Jitter?

- Phase Noise
- Voltage Noise + Threshold



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Relating Phase Noise and Jitter

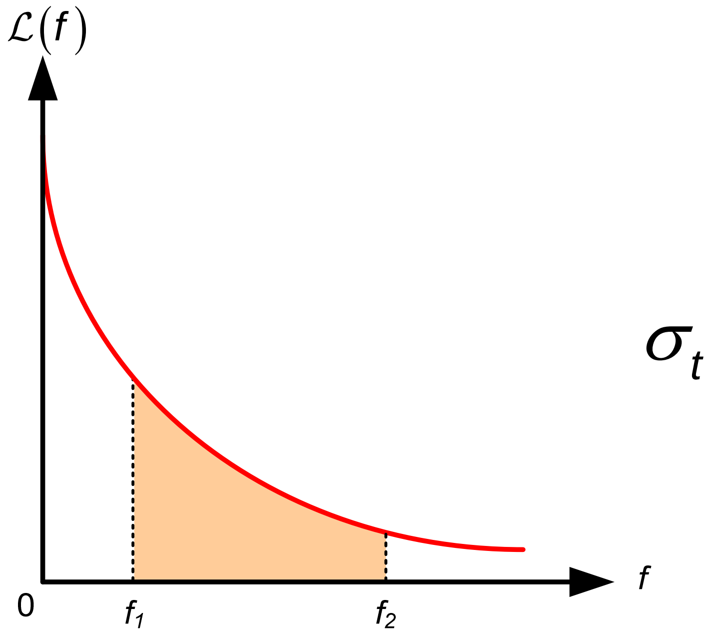
$$j(t) = \frac{\varphi(t)}{2\pi \cdot f_c}$$

$$\mathcal{L}(f) = 10 \cdot \log\left[\frac{1}{2} S_\varphi(f)\right]$$

$$\sigma_t = \sqrt{\frac{1}{T}\int_0^T j^2(t)\,dt} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{\varphi(t)}{2\pi f_c}\right)^2 dt} = \frac{1}{2\pi f_c}\sqrt{\int_{f_1}^{f_2} S(f)\,df}$$

$$\sigma_t = \frac{1}{2\pi f_c}\sqrt{2\int_{f_1}^{f_2} 10^{\frac{\mathcal{L}(f)}{10}} \cdot df}$$

Relating Phase Noise and Jitter



$$\sigma_t = \frac{1}{2\pi f_c}\sqrt{2\int_{f_1}^{f_2} 10^{\frac{\mathcal{L}(f)}{10}} \cdot df}$$

Effects of Jitter

- Increase in system noise
- Uncertainty in the actual phase of the sample
- Intersymbol interference

Increase in System Noise

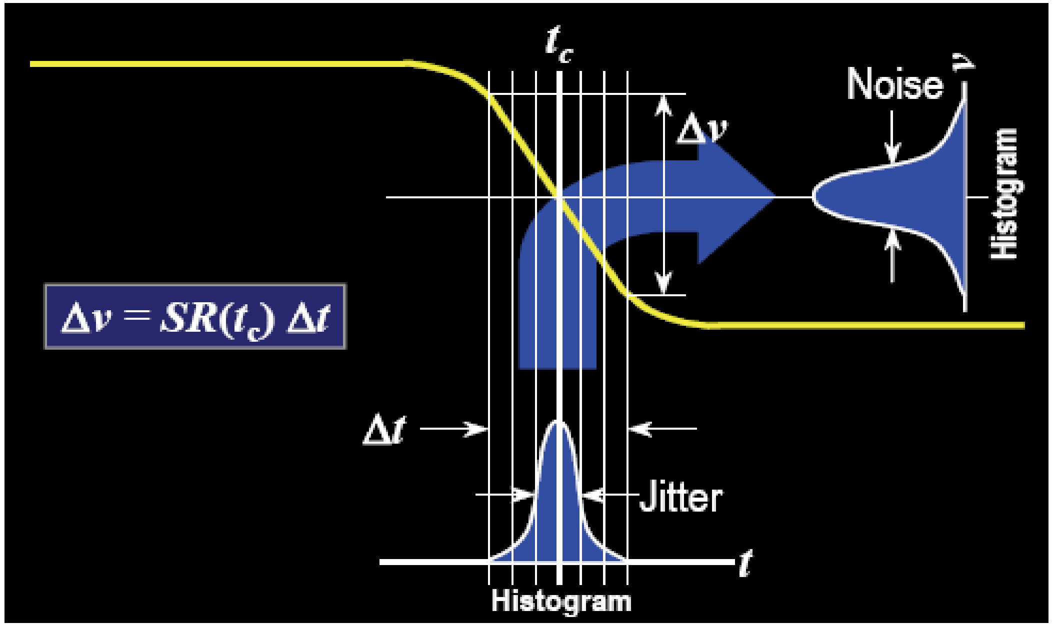
$$v(t) = A \cdot \sin(2\pi \cdot f \cdot t)$$

$$\frac{d}{dt} v(t) = 2\pi \cdot f \cdot A \cdot \cos(2\pi \cdot f \cdot t)$$

$$\left. \frac{d}{dt} v(t) \right|_{\max} = 2\pi \cdot f \cdot A \quad \rightarrow \quad \text{maximum slew rate}$$

Volate Error = Slew Rate × Timing Jitter

Sampling Clock With Jitter



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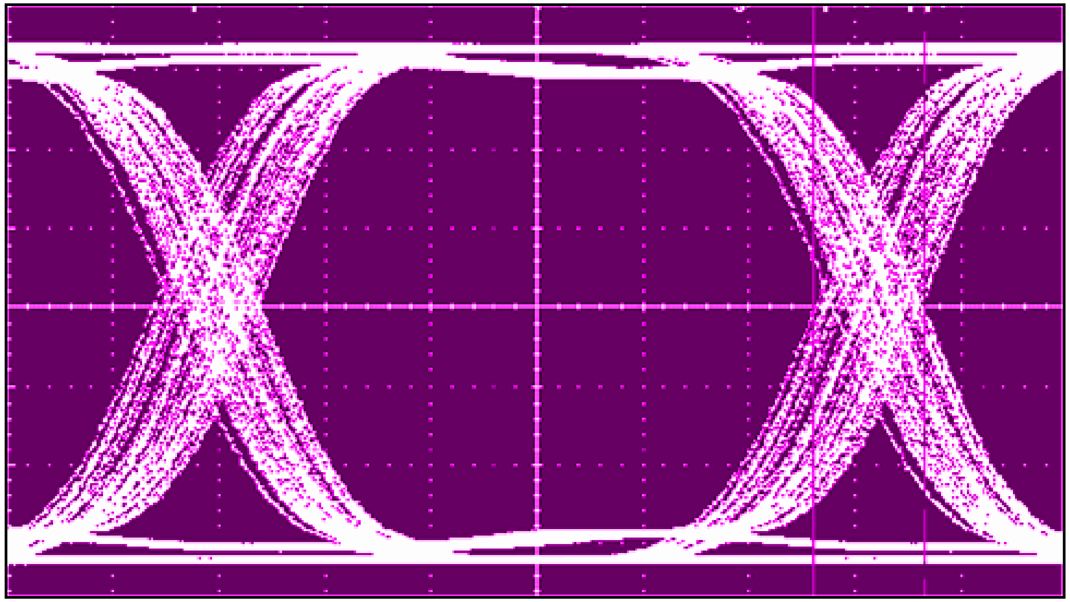
Eye Diagram



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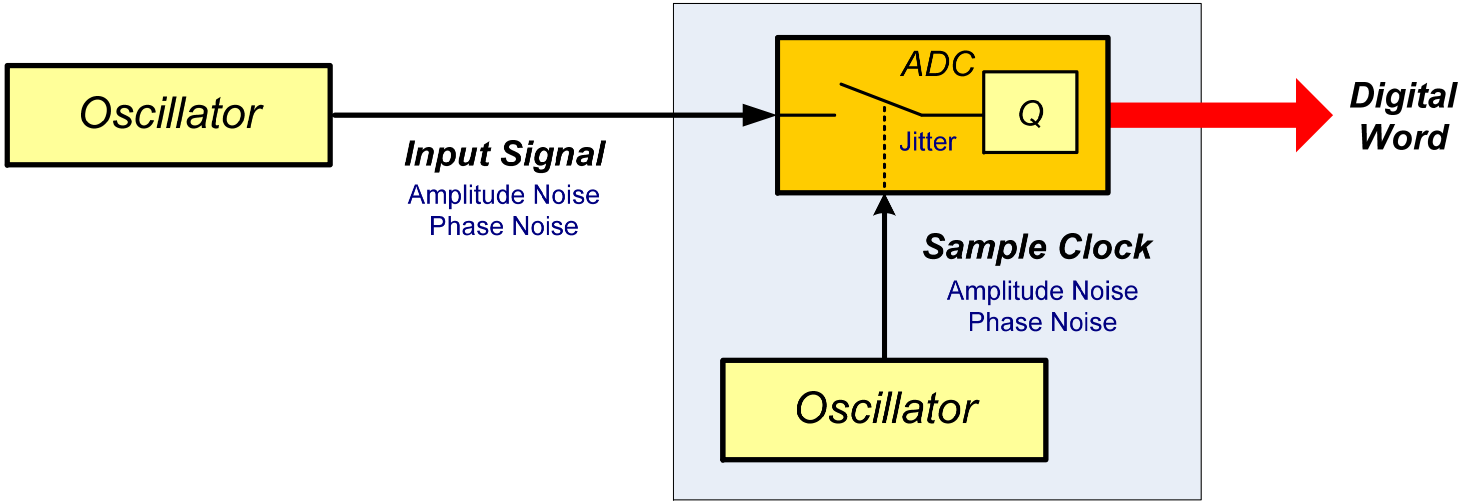
Measuring Jitter

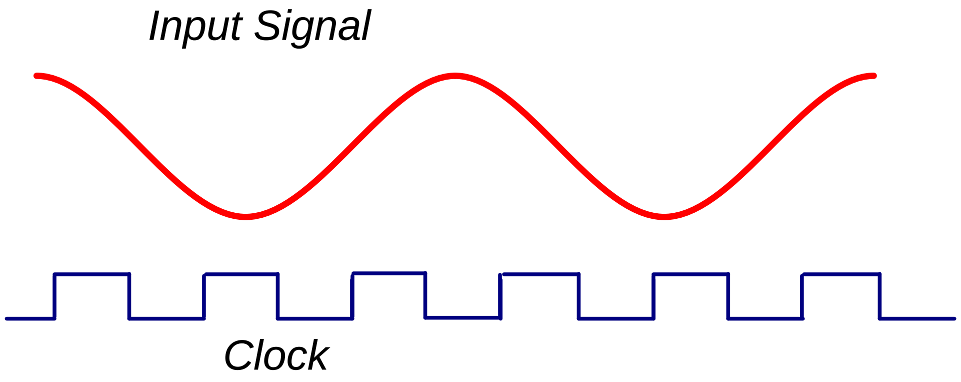
Jitter Sources



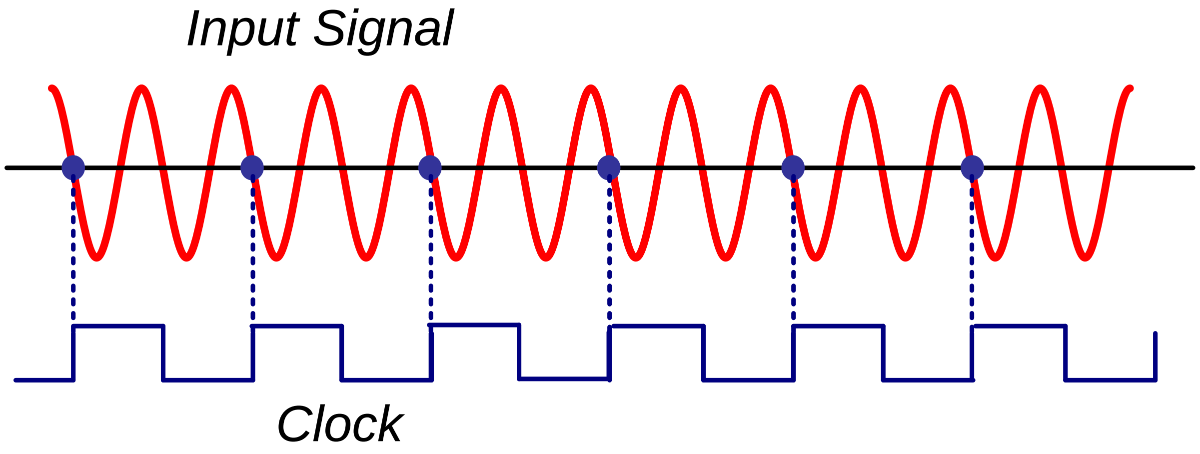
Measured Noise

- The voltage noise is due to
  - Amplitude noise
  - Jitter (through signal slew rate)
    - Input signal phase noise (through input signal frequency)
    - Clock signal phase noise (through clock frequency)
    - Sampling circuit jitter

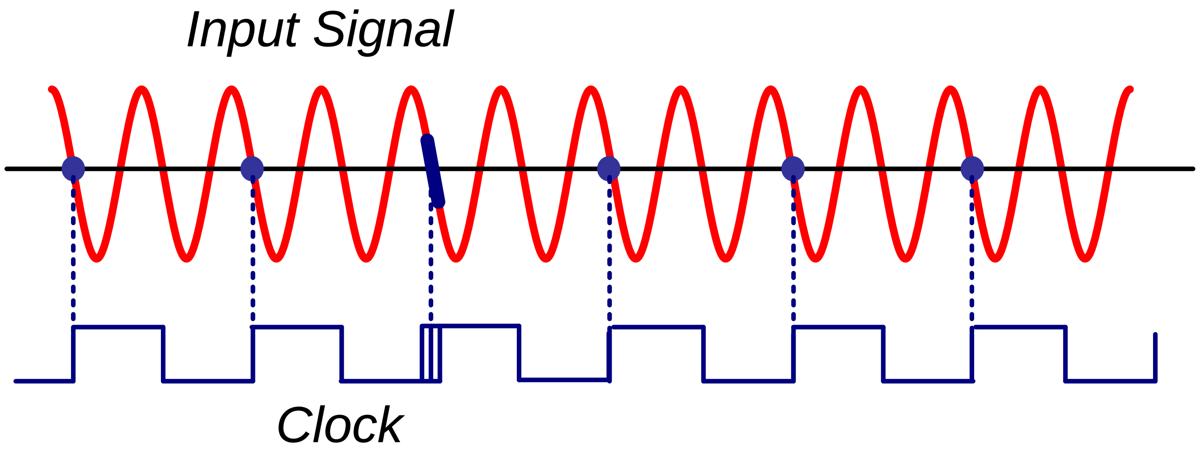
Locked Histogram Test



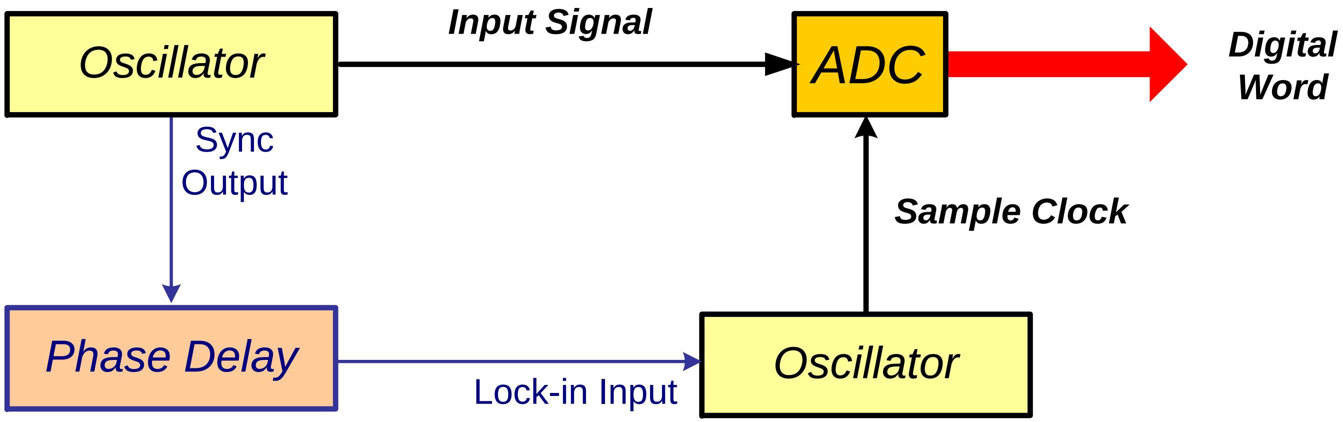
Locked Histogram Test



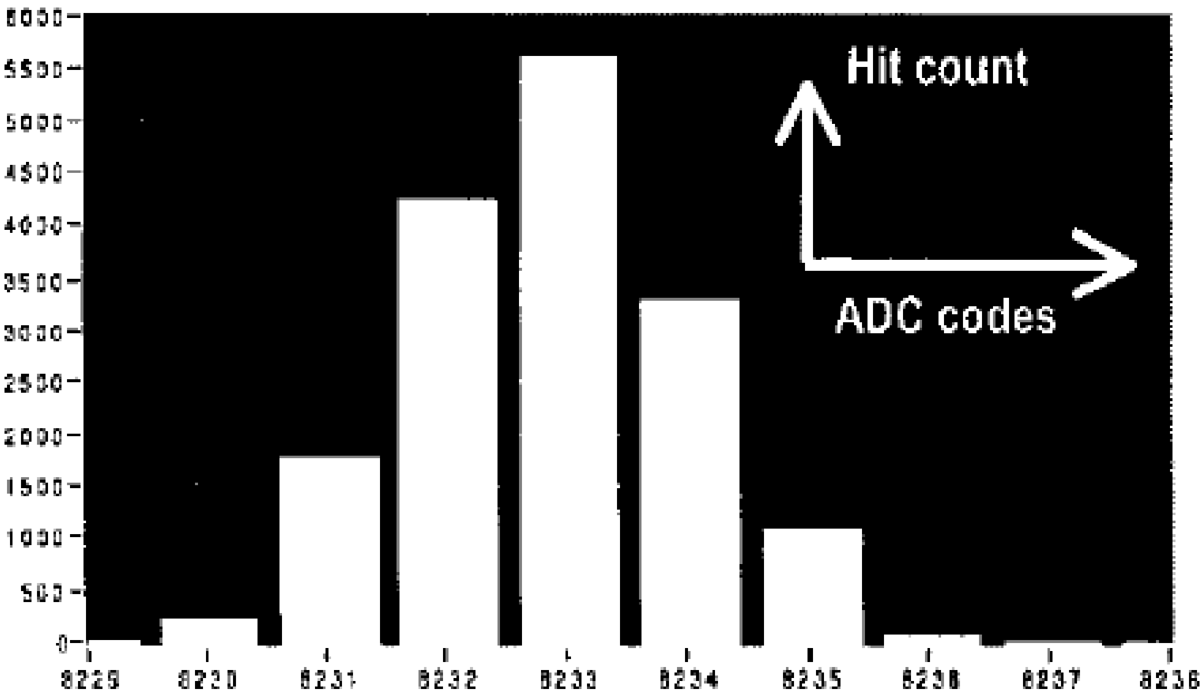
Locked Histogram Test



Circuit


Oscillator
Input Signal
ADC
Digital Word
Sync Output
Sample Clock
Phase Delay
Lock-in Input
Oscillator

Output Codes Histogram



Locked Histogram Test

$$v(t) = A \cdot \cos(2\pi f \cdot t)$$

$$\left| \frac{d}{dt} v(t) \right| = 2\pi \cdot f \cdot A \cdot \sin(2\pi \cdot f \cdot t)$$

$$\left| \frac{d}{dt} v(t) \right|_{v(t)=0} = 2\pi \cdot f \cdot A$$

$$\boxed{\sigma_t = \frac{\sigma_v}{2\pi f A}}$$

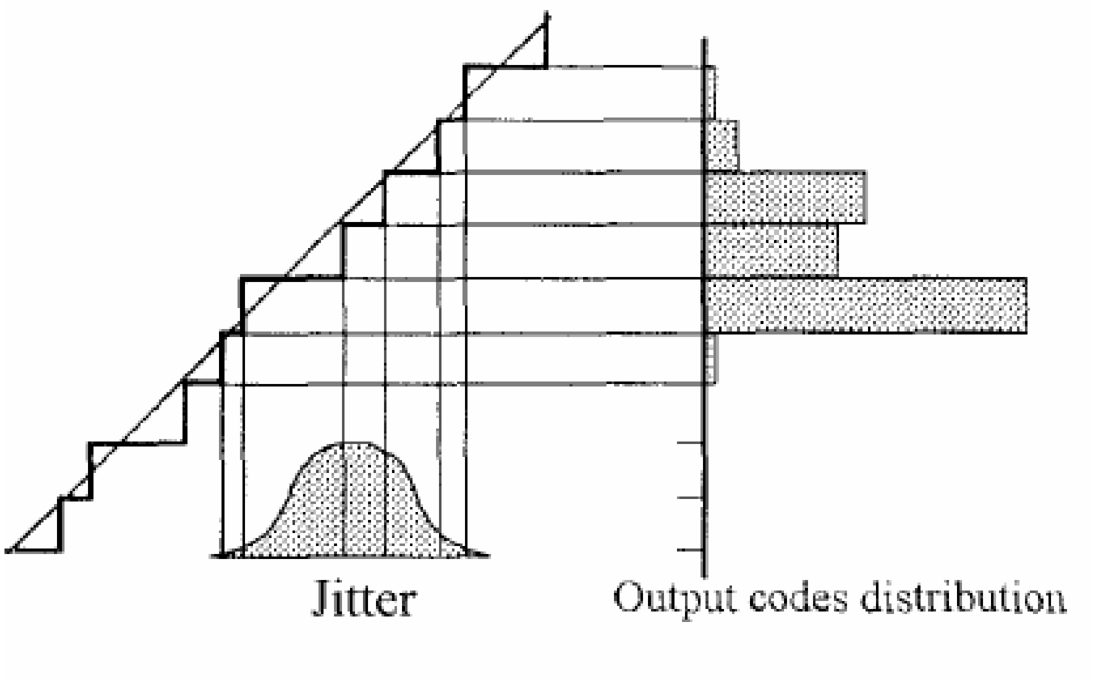
Input Frequency Choice

- Input frequency has to be a multiple of clock frequency
- Input frequency should be as high as possible
- High input frequencies will induce nonlinearities in the ADC

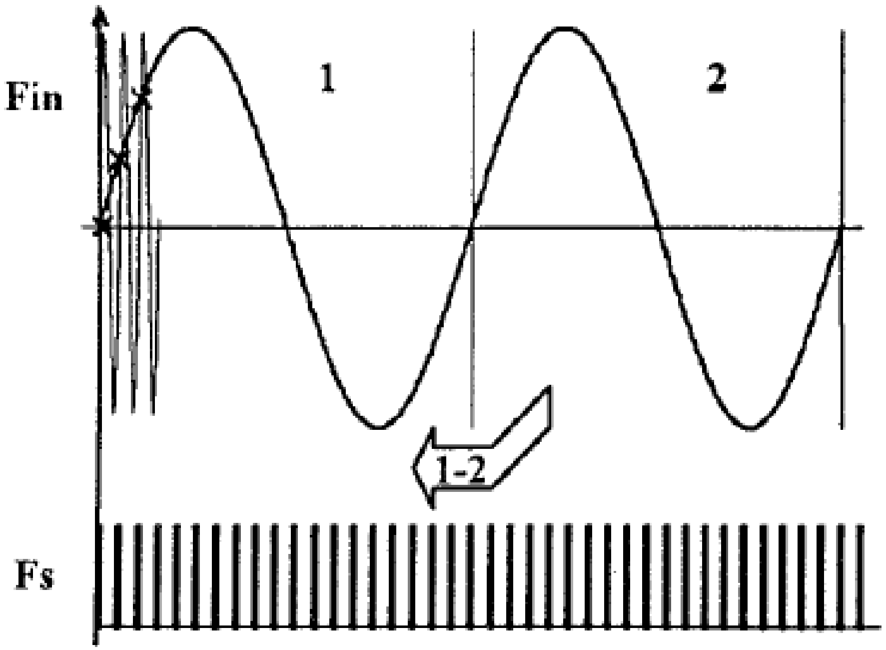
Problems

- Phase delay is needed for the sample clock but it is not accurate enough
- Voltage noise is measured together with jitter
- Differential phase noise between synthesizers is included in the result

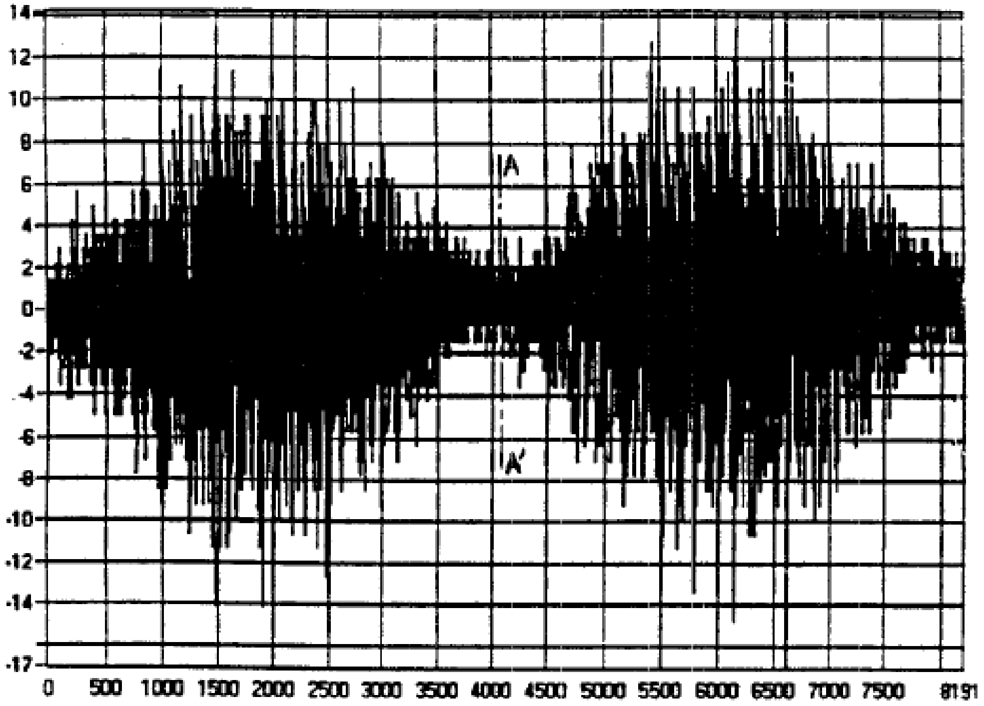
Effect of ADC Non Linearity



Double Beat Frequency Method



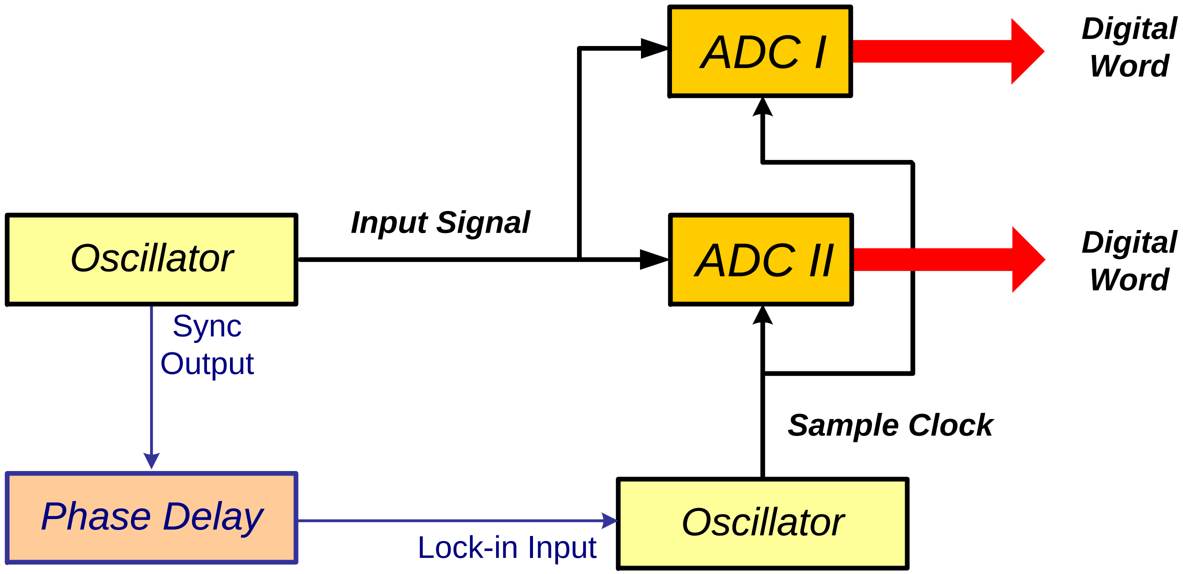
Double Beat Frequency Method



Advantages

- Eliminates the signal and systematic errors
  - Differential Non Linearity
  - Quantization Noise

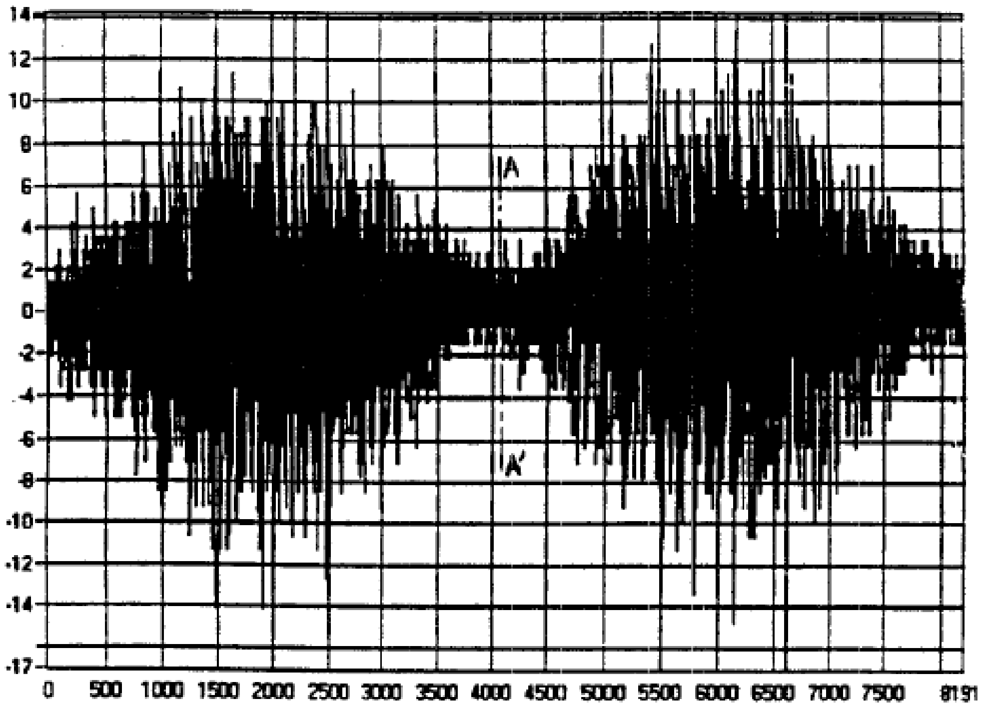
Removing Differential Phase Noise



Procedure

- Subtract the Amplitude Noise obtained in each ADC
- We get the RMS noise of each ADC quadratically added (power added)

Separating Amplitude Noise and Jitter induced Voltage Noise



From Voltage Noise to Jitter

- Divide by the signal slope
- Filter signal and keep 5 harmonics to estimate signal slope

Separating The Jitter From Both ADCs

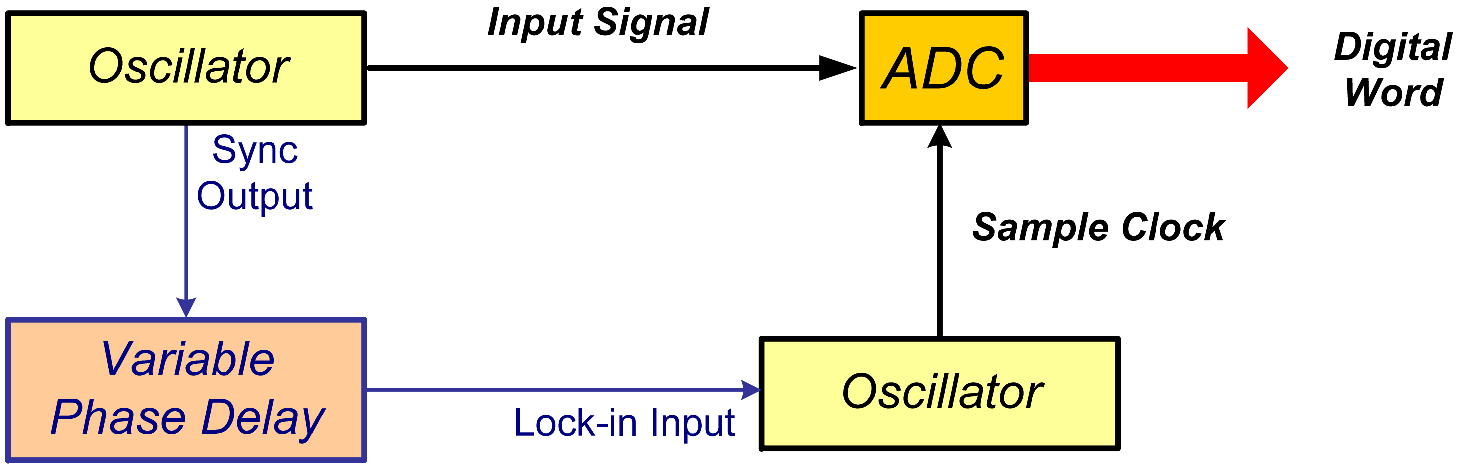
- Use 3 identical ADCs
- Use 2 at a time – 3 Combinations
- From the 3 measurements (σ_{t12}, σ_{t13} and σ_{t13}) compute the jitter of each ADC

$$\sigma_{t12}^2 = \sigma_{t1}^2 + \sigma_{t2}^2$$

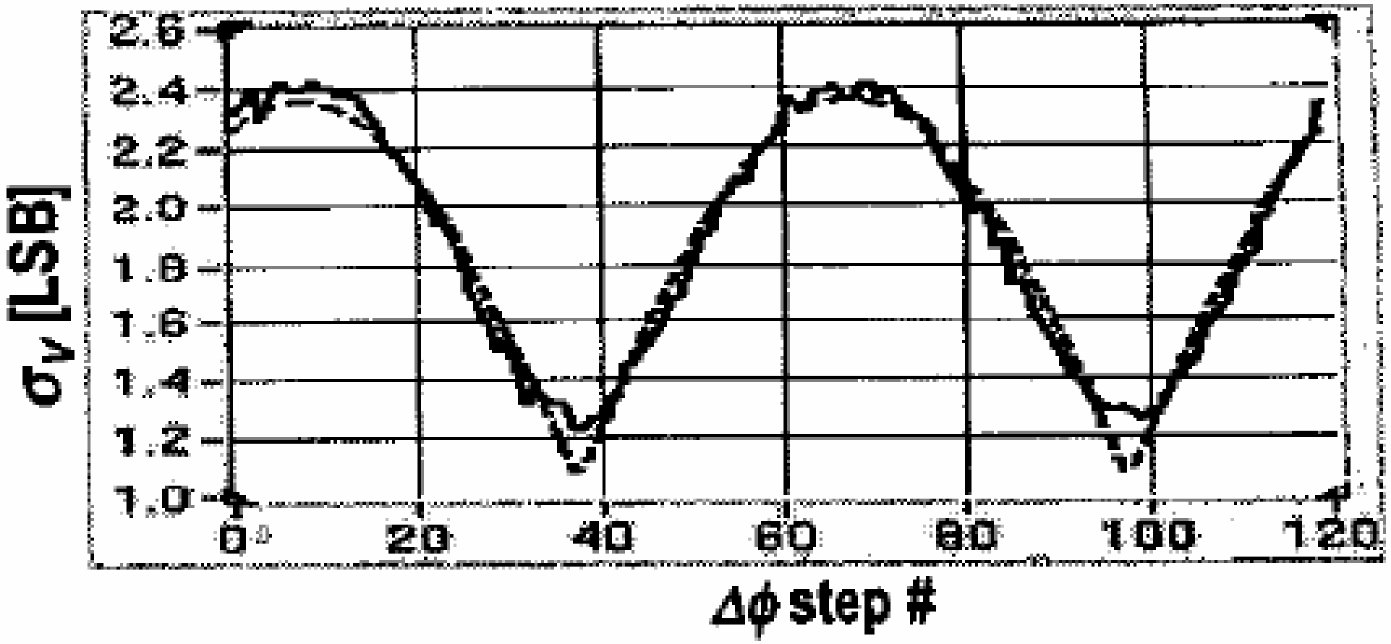
$$\sigma_{t13}^2 = \sigma_{t1}^2 + \sigma_{t3}^2$$

$$\sigma_{t23}^2 = \sigma_{t2}^2 + \sigma_{t3}^2$$

Variable Phase Delay



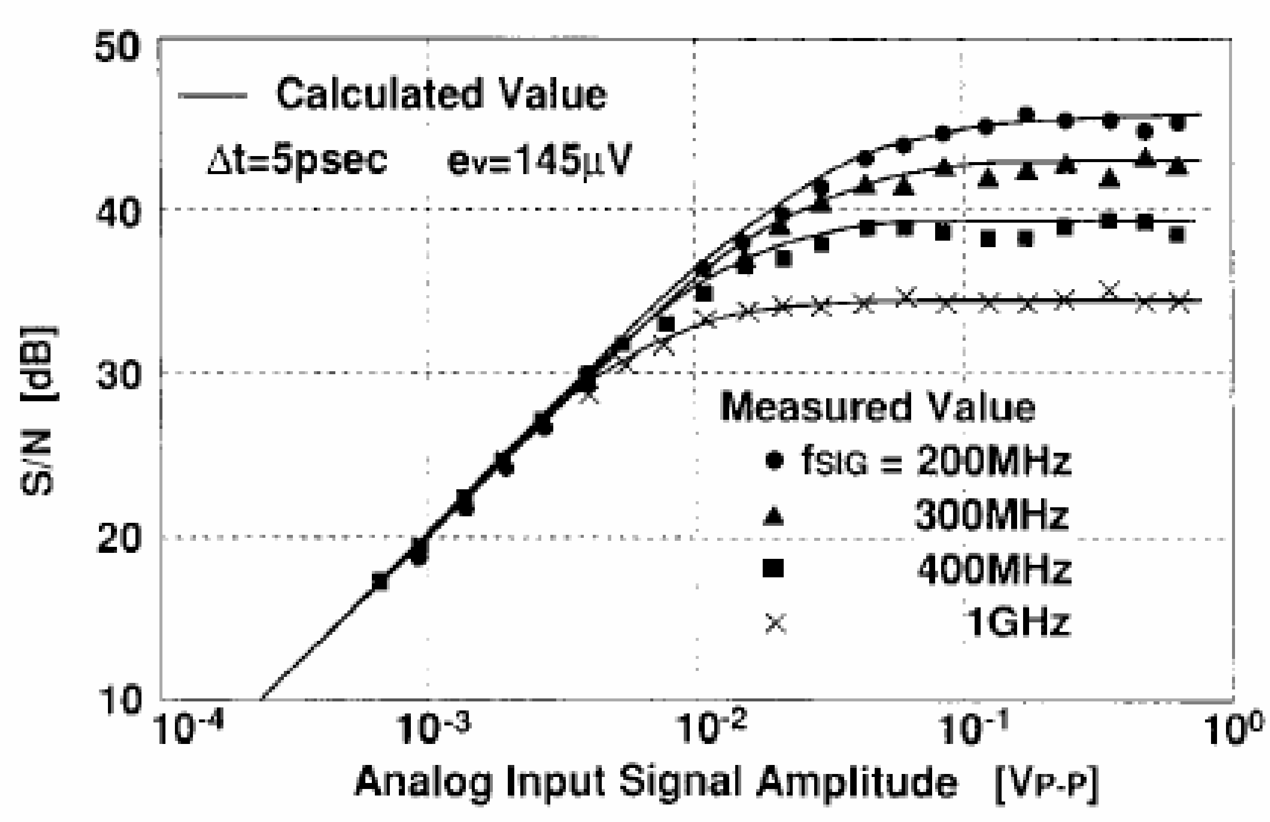
Locked Histogram with Different Delays



Fitting

$$\sigma_v^2 = \sigma_t^2 \left| \frac{d}{dt} v(t) \right|^2 + \sigma_n^2 = \sigma_j^2 \omega^2 A^2 \sin^2(\omega t) + \sigma_n^2$$

Jitter from SNR



Effect of Jitter on the RMS Voltage of a Sine Wave Residuals

$$\sigma_v = \left.\frac{dv}{dt}\right|_{rms} \cdot \sigma_t = \sqrt{\frac{1}{T}\int_0^T \left(\frac{dv(t)}{dt}\right)^2 dt} \cdot \sigma_t$$

$$\sigma_v = \sqrt{2}\pi f A \sigma_t$$

Influence of Jitter on SNR

$$SNR = 20\log\left(\frac{\frac{A}{\sqrt{2}}}{\sqrt{\left(\sqrt{2}\pi Af\right)^2 \sigma_t^2 + \sigma_n^2}}\right)$$

SNR

- Changing the input frequency will change the contribution of jitter to SNR
- Changing the sine wave amplitude will change the contribution of amplitude noise

$$SNR = -20\log\sqrt{(2\pi f)^2\,\sigma_t^2 + 2\frac{\sigma_n^2}{A^2}}$$

Different Jitter Sources

- ADC sampling
- Input signal
- Sample clock

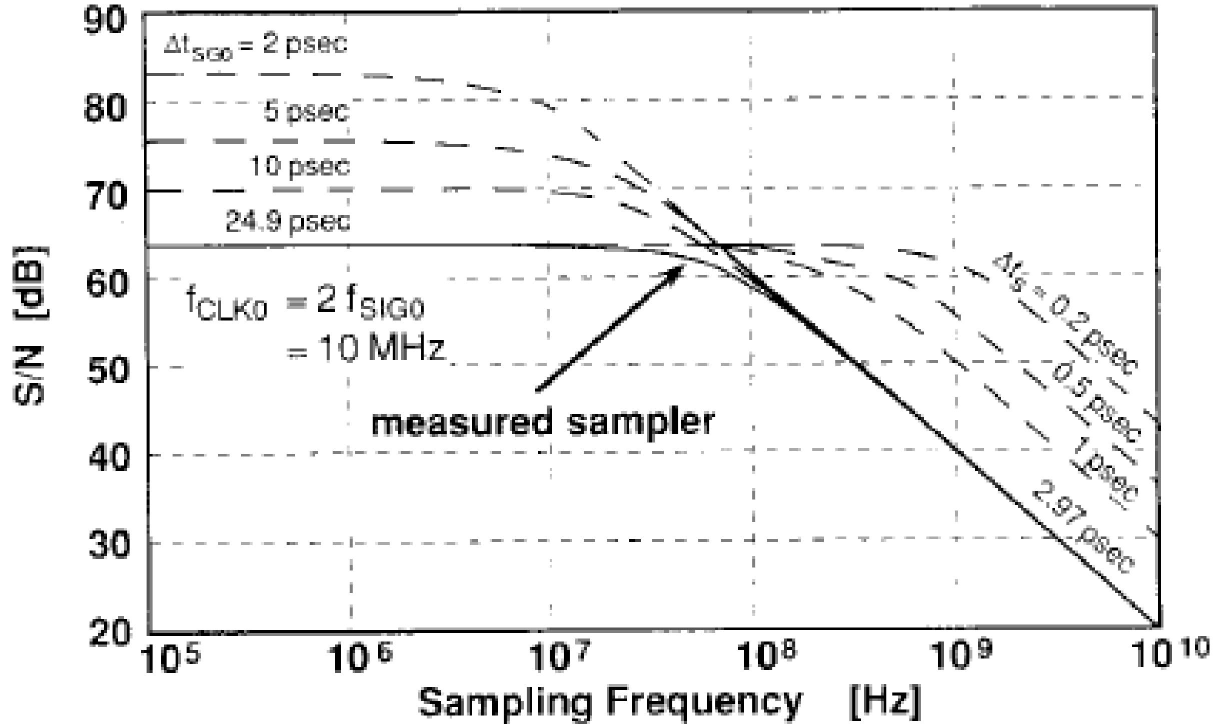
$$\sigma_t^2 = \sigma_S^2 + \sigma_{SIG}^2 + \sigma_{CLK}^2$$

Separating Jitter Sources

- Input signal jitter depends on the input signal amplitude
- Clock signal depends on clock frequency

$$\sigma_t^2 = \sigma_S^2 + \frac{f_{SIG0}^2}{f_{SIG}^2}\sigma_{SIG0}^2 + \frac{f_{CLK0}^2}{f_{CLK}^2}\sigma_{CLK0}^2$$

Distinguishing Jitter Source



Jitter Source Test

- Repeat the test for 3 different sets of input and clock frequency

f_{CLK} [MHz]	f_{SIG} [MHz]	Δt[psec]
20	40	4.99
20	20	7.26
10	40	6.74

IEEE Std 1057

- Large sine wave
- Asynchronous triggering
- Sinefitting
- Divide residuals by signal derivative (remove low derivative values)
- Collect 10 records
- Average them point by point to eliminate
  - Amplitude noise
  - Quantization noise
  - Harmonic distortion

IEEE Std 1057

- Compute the mean of all points
- The difference of the errors from the mean are the fixed errors in sample time
- The standard deviation is the aperture uncertainty

IEEE Std 1057 – Alternate Method

- Acquire two records at different frequencies
- Calculate RMS error from residuals of sine fitting
- Calculate jitter:

$$\sigma_t = \frac{\sqrt{\sigma_B^2 - \sigma_A^2}}{\sqrt{2}\pi A f_B}$$

Conclusion

- Phase noise depends on the oscillator used
- Jitter is caused by phase noise (in the time domain) and by sampling circuits
- Jitter and its sources are measured by taking into account the influence of frequency and amplitude of the signals used